Name: _____

Problem Set 5
Math 4281, Fall 2016
Due: Wednesday, October 12

Complete the following items, staple this page to the front of your work, and turn in your assignment at the beginning of class on the due date. Remember to list your collaborators on each problem.

The complex numbers

1. Prove the following properties of the modulus of a complex number. Let $z, w \in \mathbb{C}$.
   a. $|zw| = |z||w|$
   b. $|\overline{z}| = |z|$
   c. $|z|^2 = z\overline{z}$
2. a. Evaluate $(4-5i) - \overline{(4i-4)}$.
   b. Convert $5\text{cis}\left(\frac{9\pi}{4}\right)$ to the form $a+bi$.
   c. Change $2+2i$ to polar coordinates.
   d. Calculate $(-i)^{10}$.
   e. Calculate $\left(\frac{1-i}{2}\right)^4$.
3. Find the sixth roots of $-3i$. Express your answers in the (exact) form $z = a+bi$ without trigonometric functions whenever possible, and then plot them in the complex plane.

Euclidean algorithm for polynomials

4. Apply the division algorithm to the polynomials $f(x), g(x) \in \mathbb{Z}_7[x]$, where
$$f(x) = x^6 + \overline{3}x^5 + \overline{4}x^2 - \overline{3}x + \overline{2} \quad \text{and} \quad g(x) = x^2 + \overline{2}x - \overline{3}.$$
   Clearly identify $q(x)$ and $r(x)$.
5. Find the greatest common divisor $d(x)$ for the polynomials $f(x), g(x) \in \mathbb{C}[x]$, where
$$f(x) = x^2 + 1 \quad \text{and} \quad g(x) = x^2 - i + 2,$$
   and find $s(x), t(x) \in \mathbb{C}[x]$ to express $d(x) = s(x)f(x) + t(x)g(x)$.
6. Show that unique factorization fails in $R[x]$ when R is not an integral domain. Consider, for example, $x^2 + x + \overline{8} \in \mathbb{Z}_{10}[x]$.

Throughout the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.
Signed: _____