Irreducible polynomials

1. Decide which of the following polynomials are irreducible in $\mathbb{Q}[x]$.
   a. $x^3 + 4x^2 - 3x + 5$
   b. $4x^3 - 6x^2 + 6x - 12$
   c. $x^4 - 180$
   d. $x^4 + x^2 - 6$
   e. $2x^4 + 3x^2 + 4x + 6$

2. The polynomial
   \[ \Phi_n(x) = \frac{x^n - 1}{x - 1} = x^{n-1} + x^{n-2} + \cdots + x + 1 \]
   is called a cyclotomic polynomial. Show that $\Phi_p(x)$ is irreducible over $\mathbb{Q}$ for any prime $p$.
   (Hint: Consider $\Phi_p(x+1)$.)

Ring homomorphisms and ideals

3. Find all ring homomorphisms:
   a. $\phi: \mathbb{Z}_2 \to \mathbb{Z}$
   b. $\phi: \mathbb{Z}_2 \to \mathbb{Z}_6$
   c. $\phi: \mathbb{Z}_6 \to \mathbb{Z}_2$

4. Prove that if $p$ is prime and $\phi: \mathbb{Z}_p \to \mathbb{Z}_p$, $\phi(a) = a^p$, is a ring homomorphism.