Name: _____

Problem Set 7
Math 4281, Fall 2016
Due: Wednesday, October 26

Complete the following items, staple this page to the front of your work, and turn in your assignment at the beginning of class on the due date. Remember to list your collaborators on each problem.

Irreducible polynomials

1. Decide which of the following polynomials are irreducible in $\mathbb{Q}[x]$.
   a. $x^3 + 4x^2 - 3x + 5$
   b. $4x^3 - 6x^2 + 6x - 12$
   c. $x^4 - 180$
   d. $x^4 + x^2 - 6$
   e. $2x^4 + 3x^2 + 4x + 6$

2. The polynomial
$$\Phi_n(x) = \frac{x^n - 1}{x - 1} = x^{n-1} + x^{n-2} + \cdots + x + 1$$
is called a *cyclotomic polynomial*. Show that $\Phi_p(x)$ is irreducible over $\mathbb{Q}$ for any prime p. (Hint: Consider $\Phi_p(x+1)$.)

Ring homomorphisms and ideals

3. Find all ring homomorphisms:
   a. $\phi\colon \mathbb{Z}_2 \to \mathbb{Z}$
   b. $\phi\colon \mathbb{Z}_2 \to \mathbb{Z}_6$
   c. $\phi\colon \mathbb{Z}_6 \to \mathbb{Z}_2$

4. Prove that if p is prime and $\phi\colon \mathbb{Z}_p \to \mathbb{Z}_p$, $\phi(a) = a^p$, is a ring homomorphism.

Throughout the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.
Signed: _____