The complex numbers

1. a. Evaluate \((4 - 5i) - (4i - 4)\).
   b. Convert \(5\text{cis}\left(\frac{9\pi}{4}\right)\) to the form \(a + bi\).
   c. Change \(2 + 2i\) to polar coordinates.
   d. Calculate \((-i)^{10}\).
   e. Calculate \((\frac{1-i}{2})^4\).

2. Find the sixth roots of \(-3i\). Express your answers in the (exact) form \(z = a + bi\) without trigonometric functions, and then plot them in the complex plane.

Euclidean algorithm for polynomials

3. Apply the division algorithm to the polynomials \(f(x), g(x) \in \mathbb{Z}_7[x]\), where
   \[ f(x) = x^6 + 3x^5 + 4x^2 - 3x + 2 \quad \text{and} \quad g(x) = x^2 + 2x - 3. \]
   Clearly identify \(q(x)\) and \(r(x)\).

4. Find the greatest common divisor \(d(x)\) for the polynomials \(f(x), g(x) \in \mathbb{C}[x]\), where
   \[ f(x) = x^2 + 1 \quad \text{and} \quad g(x) = x^2 - i + 2, \]
   and find \(s(x), t(x) \in \mathbb{C}[x]\) to express \(d(x) = s(x)f(x) + t(x)g(x)\).

5. Show by example that unique factorization fails in \(R[x]\) when \(R\) is not an integral domain. For instance, consider \(x^2 + x + \bar{8} \in \mathbb{Z}_{10}[x]\).

Throughout the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ________________________________