1. Prove Theorem 2 in the Direct and Inverse Limits section of your notes.

**Thm 2.** Let \( \{M_i, f_{ji}\}, \{M'_i, f'_{ji}\}, \) and \( \{M''_i, f''_{ji}\} \) be three direct systems of \( R \)-modules over the same directed set \( I \) and suppose that we have exact sequences

\[
M'_i \xrightarrow{u_i} M_i \xrightarrow{v_i} M''_i
\]

where \( u_i \) and \( v_i \) are compatible with the maps in the direct system. Then

\[
\lim_i M'_i \xrightarrow{\lim u_i} \lim_i M_i \xrightarrow{\lim v_i} \lim_i M''_i
\]

is also exact.

2. Let \( R \) be a ring, \( x \in R \), \( M \) and \( R \)-module, and \( \{M_i, f_{ij}\} \) the direct system of \( R \)-modules over the index set \( \mathbb{N} \) given by

\[
M_1 = M \xrightarrow{x} M_2 = M \xrightarrow{x} M_3 = M \xrightarrow{x} \cdots .
\]

Show that there is an \( R \)-isomorphism \( \lim_i M_i \cong M_x \) mapping \( \alpha_i(a) \) to \( \frac{a}{x^i} \).

3. Prove Theorem 6.a in the Direct and Inverse Limits section of your notes.

**Thm 6.** Let \( \{M_i, f_{ij}\}, \{M'_i, f'_{ij}\}, \) and \( \{M''_i, f''_{ij}\} \) be inverse systems of \( R \)-modules over \( I = \mathbb{N} \). Let \( u_i : M'_i \rightarrow M_i \) and \( v_i : M_i \rightarrow M''_i \) be \( R \)-linear maps compatible with the maps in the inverse systems.

a. If \( 0 \rightarrow M'_i \xrightarrow{u_i} M_i \xrightarrow{v_i} M''_i \) are exact, then eventually

\[
0 \rightarrow \lim_i M'_i \xrightarrow{\lim u_i} \lim_i M_i \xrightarrow{\lim v_i} \lim_i M''_i
\]

is exact.

4. Eisenbud, Exercise 7.11.

5. Let \( R \) be a Noetherian ring. Show that \( R[[x_1, \ldots, x_n]] \) is a Noetherian ring.