

LOCAL COHOMOLOGY IN COMMUTATIVE ALGEBRA AND ALGEBRAIC GEOMETRY

TALK ABSTRACTS

Speaker: **Josep Àlvarez Montaner**

Title: ***D*-modules, Bernstein-Sato polynomials and F-invariants of direct summands**

Abstract: In this joint work with Craig Huneke and Luis Núñez-Betancourt we study structural properties of *D*-modules over a direct summand of a polynomial ring or a formal power series ring with coefficients over a field. We prove that localizations and local cohomology modules have finite length as a *D*-module and we show the existence of a Bernstein-Sato polynomial in this non-regular framework. Time permitting, we will also discuss some results on F-jumping numbers and F-thresholds for this family of rings.

Speaker: **Lucho Avramov**

Title: Perfect complexes, phantom maps, and intersection theorems

Abstract: The existence of big Cohen-Macaulay modules was proved in equal characteristics by Hochster (in 1973) and in mixed characteristics by Y. Andre (in 2016). During the 1970s and 1980s, Hochster and others deduced from their existence several conjectures concerning bounded complexes of finite free modules over local rings. Concurrently and independently, such complexes were being studied under the name of perfect complexes in the context of derived categories of modules. It will be shown how big Cohen-Macaulay modules can be used to obtain information on the structure of derived categories. This leads to new proofs and to some stronger forms of classical intersection theorems in local algebra, and also to relations to still open problems, such as the Buchsbaum-Eisenbud-Horrocks Conjecture. The talk is based on joint work with Srikanth Iyengar and Amnon Neeman.

Speaker: **Margherita Barile**

Title: **Answering a question on arithmetical rank**

Abstract: We give a partial answer to an open question regarding the arithmetical rank of determinantal ideals.

Speaker: **Bhargav Bhatt**

Title: **Perfect(oid) rings**

Abstract: A characteristic p ring is called perfect if its Frobenius map is an isomorphism. Despite their largeness, these rings have some remarkably nice algebraic properties. I will discuss some of these features, and use them to give a simple proof of Kunz's characterization of regularity as the flatness of Frobenius (based on joint work with Scholze). I will then

explain the analog of this story in mixed characteristic via perfectoid rings (joint work with Iyengar and Ma).

Speaker: **Holger Brenner**

Title: **Tight closure and continuous closure**

Abstract: In joint work with Jonathan Steinbuch, we show that for a primary ideal in a normal domain of finite type over \mathbb{C} , tight closure is contained in continuous closure. This rests on results of Huneke and Vraciu on special tight closure, the notion of axes closure, reduction to positive characteristic and a result of Hochster and Epstein.

Speaker: **John Greenlees**

Title: **Rings with a twisted local cohomology theorem**

Abstract: The speaker and Lyubeznik (2000) studied graded rings with a local cohomology theorem with shift v . For a commutative, connected graded k -algebra R_* for a field k this means that there is a spectral sequence

$$H_m^*(R_*) \Rightarrow \Sigma^v \text{Hom}_k(R_*, k).$$

Of course Gorenstein rings are examples, but there are many others. such as the group cohomology ring $R_* = H^*(G; k)$ for a finite group G . Indeed, these come up often as the coefficient rings of equivariant cohomology rings. We showed for example that these rings localize (so that, for example the LCTv rings are Gorenstein in codimension 0).

The talk is about a new class of examples where the grading is not over the integers but rather over the real representation ring of G . The point is that the shift v is not an integer, so that one must consider the more general grading. The first example is ordinary integral cohomology, and the second is Atiyah's K-theory with reality. This will draw on joint work with Lennart Meier.

Speaker: **Daniel Hernández**

Title: **Generalized Frobenius powers**

Abstract: In this talk, we introduce the notion of generalized Frobenius powers of ideals in a regular ring. As an application, we demonstrate a prime characteristic analog of Howald's result relating the log canonical threshold of a polynomial with that of its term ideal. Furthermore, via generalized Frobenius powers of monomial ideals, we establish certain uniform behavior of a polynomial over the rational numbers under the process of reduction to large prime characteristic. This is joint work with Emily Witt and Pedro Texeira.

Speaker: **Jen-Chieh Hsiao**

Title: **Strong Monodromy Conjecture for Monomial Ideals on Toric Varieties**

Abstract: We show that the real part of a pole of Denef–Loeser's motivic zeta function associated to a monomial ideal on an affine toric variety is a root of the corresponding Bernstein–Sato polynomial defined in a joint work with Matusevich. This can be viewed as

a toric analogue of Igusa's strong Monodromy Conjecture, which was resolved by Howald–Mustață–Yuen for the case of monomial ideals on affine spaces.

Speaker: **Mel Hochster**

Title: **Thirteen Open Questions about Local Cohomology**

Abstract: The talk will discuss a number of open questions: for many of them, Gennady Lyubeznik raised the question and made the first progress. Some of the other questions are connected with the local homological conjectures or the theory of content of local cohomology, studied by the speaker jointly with Craig Huneke and, in a recent paper, with Wenliang Zhang, and are related to the problem of extending tight closure theory to mixed characteristic.

Speaker: **Craig Huneke**

Title: **Refinements of a classical inequality of Lech**

Abstract: This work, which is joint with Ananth Hariharan and Javid Validashti, concerns an old inequality of Lech which states that if R is a Noetherian local ring of dimension d and multiplicity e , and I is an ideal of finite colength m , then the multiplicity of I is bounded above by $d!em$.

Our work began as an attempt to refine this inequality to one which will be sharper, by relying on the asymptotic properties of the inequality as we replace I by I^n . We are able to prove the new inequality in several cases, and it also suggests other improvements which are interesting. This talk will give background information, the new inequality, several interesting examples, and a summary of what we know.

Speaker: **Mordechai Katzman**

Title: **F -modules: digging at the roots**

Abstract: In this talk I describe constructive methods for extracting information about F -finite F -modules by analyzing their roots or, better still, their generating morphisms. The first such method gives a bound for the length of F -finite F -modules, which is constructed from the set of prime special ideals introduced by R.Y. Sharp and the second is a method for computing the support of F -finite F -modules from a generating morphism without the need to compute roots. (joint work with W. Zhang, L. Ma and I. Smirnov).

Speaker: **Anton Leykin**

Title: **Beyond Polyhedral Homotopies**

Abstract: We present a new algorithmic framework which utilizes tropical geometry and homotopy continuation for solving systems of polynomial equations where some of the polynomials are generic elements in linear subspaces of the polynomial ring. This approach generalizes the polyhedral homotopies by Huber and Sturmfels.

Speaker: **Linquan Ma**

Title: **Perfectoid test ideals**

Abstract: Inspired by the recent solution of the direct summand conjecture of Andre and Bhatt, we introduce perfectoid multiplier/test ideals in mixed characteristic. As an application, we obtain a uniform bound on the growth of symbolic powers in regular rings of mixed characteristic analogous to results of Ein-Lazarsfeld-Smith and Hochster-Huneke in equal characteristic. This is joint work with Karl Schwede.

Speaker: **Kazuma Shimomoto**

Title: **Hyperplane sections of normal schemes in mixed characteristic**

Abstract: This is joint work with J. Horiuchi (Nippon Institute of Technology). A classical theorem of Bertini states that projective varieties share many good properties with their general hyperplane sections. Various forms of Bertini-type results are known to hold over rings and schemes over a field. In this talk, we discuss the mixed characteristic case. The main result has an application to the restriction map (Gysin map) of Weil divisor class groups.

Speaker: **Anurag Singh**

Title: **Hankel determinantal rings**

Abstract: We will discuss various aspects of rings defined by minors of Hankel matrices of indeterminates, including a proof that these have rational singularities. This is part of work in progress with Aldo Conca, Maral Mostafazadehfard, and Matteo Varbaro.

Speaker: **Nicholas Switala**

Title: **The de Rham cohomology of graded D -modules over polynomial rings**

Abstract: Recent work of Hartshorne-Polini and Lyubeznik relates the de Rham cohomology of a holonomic D -module M over a formal power series ring R (in characteristic zero) to the Ext modules $Ext_D^*(M, E)$ where E is the top local cohomology module of R . In joint work with Wenliang Zhang, we establish analogous results for graded D -modules over polynomial rings. Along the way, we prove that if M is a graded holonomic D -module, its graded Matlis dual is again a graded D -module whose de Rham cohomology is k -dual to that of M .

Speaker: **Emily Witt**

Title: **Connectedness and Lyubeznik numbers**

Abstract: The second vanishing theorem of local cohomology, due to Ogus and Peskine-Szipro, characterizes the connectedness of the punctured spectrum via the vanishing of local cohomology. Moreover, results of Hochster-Huneke, Lyubeznik, and Zhang establish that Lyubeznik numbers, numerical invariants defined using local cohomology, determine whether or not the spectrum of a ring is highly connected. In this talk, we discuss how Lyubeznik numbers encode more refined information about connectedness properties of spectra. This is joint work with Luis Núñez-Betancourt and Sandra Spiroff.