LOCAL COHOMOLOGY IN COMMUTATIVE ALGEBRA AND ALGEBRAIC GEOMETRY

POSTER ABSTRACTS

Presenter: **Eric Canton**
Title: **Asymptotic invariants of ideal sequences in positive characteristic via Berkovich spaces**
Abstract: We present some recent progress in the study of asymptotic invariants of graded sequences of ideals on smooth varieties in positive characteristic. Specifically, we show that for sequences of monomial ideals vanishing at the origin of affine space, there is always an Abhyankar valuation minimizing the log canonical threshold of the sequence, and this valuation must be monomial for some choice of coordinates. Jonsson and Mustaţă deduce this in characteristic zero using resolutions of singularities; we gain the needed structure in positive characteristics using Frobenius splitting methods, with the occasional input from local uniformization of Abhyankar valuations.

Presenter: **Javier A. Carvajal-Rojas**
Title: **Finite torsors over strongly F-regular singularities**
Abstract: We discuss an extension to the work by K. Schwede, K. Tucker and myself on the étale fundamental group of strongly F-regular singularities. Concretely, we study the existence of torsors over the regular locus that do not come from restricting a torsor over the whole spectrum. In the abelian case, these torsors naturally relate to the action of Frobenius on local cohomology. We generalize to this context the transformation rule for the F-signature we had in our study of the étale fundamental group. Using these two ingredients, we prove that every strongly F-regular singularity is dominated by one satisfying that abelian torsors over its own regular locus are restrictions of torsors everywhere. In the process, we will demonstrate that cyclic covers over strongly F-regular (resp. F-pure) local rings stay strongly F-regular (resp. F-pure). As an application, we bound the torsion of the Picard group of a strongly F-regular singularity by the reciprocal of its F-signature.

Presenter: **Rankeya Datta**
Title: **Uniform approximation of Abhyankar valuation ideals in function fields of prime characteristic**
Abstract: Using the theory of asymptotic test ideals, we prove the prime characteristic analogue of a characteristic 0 of Ein, Lazarsfeld and Smith on uniform approximation of valuation ideals associated to real-valued Abhyankar valuations centered on regular varieties over perfect fields.

Presenter: **Brent Holmes**
Title: **A Generalized Serre Condition**
Abstract: A ring satisfies Serre's condition (S_ℓ) if for all $P \in \operatorname{Spec} R$,

$$\operatorname{depth} R_P \geq \min\{\ell, \dim R_P\}.$$

Serre's condition has been a topic of expanding interest. In this poster, we examine a generalization of Serre's condition (S_ℓ^j). We say a ring satisfies (S_ℓ^j) when $\operatorname{depth} R_P \geq \min\{\ell, \dim R_P - j\}$ for all $P \in \operatorname{Spec} R$. We prove generalizations of results for rings satisfying Serre's condition.

Presenter: **Jennifer Kenkel**
Title: **Local Cohomology of Thickenings**
Let R be a standard graded polynomial ring that is finitely generated over a field, let m be the homogeneous maximal ideal of R, and let I be a homogeneous prime ideal of R. A recent paper of Bhatt, Blickle, Lyubeznik, Singh, and Zhang examined the local cohomology of the thickenings R/I^t in char 0, and provided a stabilization result for these cohomology modules. In my thesis research, I have been considering the corresponding question in positive characteristic, where the answer turns out to be remarkably different. I have also been analyzing, in greater depth, the characteristic zero results for thickenings of determinantal varieties.

Presenter: **Patricia Klein**
Title: **A Generalization of a Theorem of Lech's**
Abstract: Let (R, m) be a complete local ring of dimension d. Lech's inequality states that for every m-primary ideal I, $\dfrac{e_I(R)}{\ell(R/I)} \leq d! \cdot e_m(R)$, where $e_I(R)$ denotes the multiplicity of I on R. We will discuss conditions on R and on R-modules M that allow us to bound $\dfrac{e_I(M)}{\ell(M/IM)}$ above and below independent of the m-primary ideal I.

Presenter: **Whitney Liske**
Title: **Defining Equations of Rees Algebras for Artinian Gorenstein Rings**
Abstract: Let $R = k[x_1, \ldots, x_d]$ be a polynomial ring in d variables over a field k. Let $m = (x_1, \ldots, x_d)$ be the maximal homogenous ideal of R. Let I be a Gorenstein ideal generated by all the generators of m^2 except for one. For each fixed d these ideals are all equivalent, up to change of coordinates. The goal is to compute the defining equations of the special fiber ring and the Rees ring of these ideals. A secondary goal is to study the algebraic properties of these blowup algebras. To compute the Rees ring, we study the Jacobian dual and the defining equations of the special fiber ring of m^2.

Presenter: **Justin Lyle**
Title: **Subcategories and Supports**
Abstract: A classical theorem of Gabriel classifies Serre subcategories of modules in terms of specialization closed subsets of $\operatorname{Spec} R$. We use supports of certain homological functors to characterize resolving subcategories of modules in terms of certain sequences of specialization closed subsets of $\operatorname{Spec} R$. We further show that the sequences consisting of closed subsets of $\operatorname{Spec} R$ are exactly those that correspond to categories which are the resolving closure of a single module.

Presenter: **Takumi Murayama**
Title: **Frobenius–Seshadri constants and characterizations of projective space**
Abstract: We introduce higher-order variants of the Frobenius–Seshadri constant due to Mustaţă and Schwede, which are defined for ample line bundles in positive characteristic. These constants are used to show that Demailly's criterion for separation of higher-order jets by adjoint bundles also holds in positive characteristic. As an application, we give a characterization of projective space using Seshadri constants in positive characteristic, which was proved in characteristic zero by Bauer and Szemberg. We also discuss connections with other characterizations of projective space.

Presenter: **Janet Page**
Title: **The Frobenius Complexity of Hibi Rings**
Abstract: Cartier algebras and their duals, rings of Frobenius operators, have come up in various contexts, especially in the study of singularities in characteristic p. When R is a complete, local ring of characteristic $p > 0$, the total Cartier algebra, which is the ring of all potential Frobenius splittings of R, is dual to the ring of Frobenius operators (p^e-linear maps) on the injective hull of the residue field. This ring of Frobenius operators need not be finitely generated over R, which led Enescu and Yao to define Frobenius complexity as a measure of its non-finite generation. In their examples, Frobenius complexity is not always even rational, but its limit as p goes to infinity is an integer. Few other examples have been computed. In this poster, I will discuss some results on the limit Frobenius complexity for Hibi rings, which are a class of toric rings defined from finite posets.

Presenter: **Michael Perlman**
Title: **Ext Modules and Regularity of Skew-Symmetric Determinantal Thickenings**
Abstract: We consider the ring $S = \mathbb{C}[x_{i,j}]_{1 \leq i < j \leq n}$ of polynomial functions on the vector space $\bigwedge^2 \mathbb{C}^n$ of complex $n \times n$ skew-symmetric matrices. The group $\mathrm{GL} = \mathrm{GL}_n(\mathbb{C})$ acts by simultaneous row and column operations on $\bigwedge^2 \mathbb{C}^n$, which induces an action on S. For every GL-invariant ideal $I \subseteq S$ and every $j \geq 0$, we describe the decomposition of the modules $\operatorname{Ext}^j_S(S/I, S)$ into irreducible GL-representations. As a consequence of our work, we give formulas for the regularity of powers and symbolic powers of ideals of Pfaffians. As another consequence, we characterize the GL-invariant ideals $I \subseteq S$ for which the induced maps

$\mathrm{Ext}^j_S(S/I, S) \to H^j_I(S)$ are injective, providing a partial answer to a question of Eisenbud-Mustaţă-Stillman.

Presenter: **Rebecca R. G.**
Title: **Test Ideals for all Characteristics**
Abstract: We define the test ideal of a general closure operation cl, and give some of its properties. If the closure cl comes from a big Cohen-Macaulay module or algebra, or a family of such modules or algebras, then the test ideal behaves similarly to the tight closure test ideal used in characteristic $p > 0$. Since big Cohen-Macaulay algebras are now known to exist in mixed characteristic, these test ideals can be used to study singularities in all characteristics.

Presenter: **Robert Roy**
Title: **Auslander-Reiten sequences for Gorenstein rings of dimension one**
Abstract: Let R be a complete (or graded-) local Gorenstein ring of dimension one, with maximal ideal m. Then using a mild additional assumption which is satisfied whenever R is a hypersurface, we show that a particular endomorphism of m produces the Auslander-Reiten sequences of Cohen-Macaulay R-modules. We have applied this result to determining the shape of some components of stable Auslander-Reiten quivers.

Presenter: **Daniel Smolkin**
Title: **Subadditivity formulas of test ideals**
Abstract: Given a ring R in characteristic p and some ideals $\mathfrak{a}$ and $\mathfrak{b}$, the *test ideal*, denoted $\tau(R, \mathfrak{a}\mathfrak{b})$ is an ideal of R that measures the singularities of R, $\mathfrak{a}$, and $\mathfrak{b}$. Hara and Yoshida showed that, when R is regular, the test ideal satisfies the *subadditivity* formula, $\tau(R, \mathfrak{a}\mathfrak{b}) \subseteq \tau(R, \mathfrak{a})\tau(R, \mathfrak{b})$. Later, Takagi proved a more general subadditivity formula, $\mathrm{Jac}(R)\tau(R, \mathfrak{a}\mathfrak{b}) \subseteq \tau(R, \mathfrak{a})\tau(R, \mathfrak{b})$. These formulas have many important applications, such as relating symbolic and ordinary powers of ideals.

We present a new subadditivity formula for test ideals using the formalism of cartier algebras and compare this formula to earlier ones. We also present some tools for computing the relevant test ideal in the toric setting.

Presenter: **Avi Steiner**
Title: **Normal A-hypergeometric D-modules via direct images**
Abstract: Let A be a $d \times n$ integer matrix, $\beta \in \mathbb{C}^n$. Previous results by Schulze and Walther give conditions on A and β such that the associated (inverse Fourier–Laplace transformed) A-hypergeometric D-module is the D-module direct image of a simple integrable connection on the torus of A. A later result by Reichelt gives a similar condition for the case of the D-module exceptional direct image. In this poster we present, for normal A, a generalization of these results: Every (inverse Fourier–Laplace transformed) A-hypergeometric system may be

constructed from a simple integrable connection by first taking an exceptional direct image and then taking a direct image.

Presenter: **Robert Walker**
Title: **Uniform Symbolic Topologies of Conic Monomial Rings**
Abstract: First, we define when a Noetherian ring has uniform symbolic topologies (USTP) on primes; all rings on the poster are reduced algebras of finite type over a field. Then we review a few signature results (constructive versus non-constructive) by pointing to concrete examples. We then define via examples what conic monomial rings are, and we include 5 chalkboard pictures to illustrate how to construct such a ring. The main result on the poster is a USTP linear algebra formula for a select class of primes inside of a conic monomial ring; we close with a few examples to illustrate the formula, where the bounds obtained are best possible.

Presenter: **Alexander York**
Title: **An Extension of a Theorem of Frobenius and Stickelberger**
Abstract: The classical structure theorem for abelian groups proved by Frobenius and Stickelberger and extended to principal ideal domains is well celebrated. By using local duality to study properties of modules in this classification, we can characterize modules of projective dimension one over a local factorial domain which have diagonalizable presentation matrices. This allows an extension of the theorem to projective dimension one over a local factorial domain.