Definitions

Each of the following definitions will be discussed during the talk, but I collect them here as a reference to potentially make the talk easier to follow.

In this talk’s we’ll be discussing Markov chains.

1. **Markov chains** can be informally described as stochastic processes where the “future is independent of the past given the present.”

   More precisely, if \( I \) is a finite or countable set then \( \{X_n\}_{n \geq 0} \) is a (time homogeneous) Markov chain taking values in \( I \) if for any choice of \( n \geq 1 \) and \( i_0, i_1, \ldots, i_n, i_{n+1} \in I \) we have

   \[
   \Pr(X_{n+1} = i_{n+1} \mid X_0 = i_0, X_1 = i_1, \ldots, X_n = i_n) = \Pr(X_{n+1} = i_{n+1} \mid X_n = i_n) = \Pr(X_1 = i_{n+1} \mid X_0 = i_n).
   \]

   A process satisfying the above property will be called **Markovian** and will have a **matrix**, \( P \), whose entries are the probabilities of transitioning between its states.

2. **Recurrent**: A Markov chain will be called recurrent if it visits each state infinitely often.

3. **Transient**: A Markov chain will be called transient if it visits each state a finite number of times.

4. **Limiting Speed**: For a random walk, the Limiting speed is given by

   \[
   \lim_{n \to \infty} \frac{X_n}{n}
   \]

   where \( X_n \) is the displacement from the origin at time \( n \).

5. In discussing Cookie Random Walks, \( M \) will be the number of cookies.

6. \( p_j \in (\frac{1}{2}, 1) \) will be referred to as the **strength of the \( j \)th cookie**.

7. The **drift**, \( \delta \), is defined as \( \delta = \sum_{j=1}^{M} (2p_j - 1) \). This is the average amount the \( M \) cookies push the walker from any given site if they’re all eaten.

8. **Hitting Time** \( T_k \): \( T_k \) will be the number of steps a walker takes to visit site \( k \) for the first time.

9. A (possibly infinite) vector \( v \) with non-negative entries summing to 1 is a **state distribution**.

10. A **stationary distribution** is a special choice of state distribution \( v \) for a Markov chain where transitioning from \( v \) will result in the same distribution of states, \( v \). These are typically denoted by \( \pi \) satisfying \( \pi P = \pi \) where \( P \) is the transition matrix for the Markov chain.

11. A special case of the Ergodic Theorem we will use: For a Markov chain \( X \) with transition matrix \( P \) if we have a stationary distribution \( \pi \) satisfying

   \[
   \pi = \lim_{n \to \infty} v P^n
   \]

   for all state distributions \( v \), then the following holds

   \[
   \lim_{n \to \infty} \frac{1}{n} \sum_{k=0}^{n-1} X_k = E\pi[X_0] = \sum_{k \geq 0} k \pi(k)
   \]