1. A friend flips 2 coins and tells you that at least one is heads. Given this, what is the probability that the first coin is heads?

2. In a town, 40% of families have a dog, 30% of families have a cat, and 25% of families that have a dog also have a cat.
   (a) What fraction of people have a dog or a cat?
   (b) What is the probability that a family with a cat has a dog?

3. $A, B,$ and $C$ are events with $P(A) = 0.3, P(B) = 0.4,$ and $P(C) = 0.5$. If $A$ and $B$ are disjoint, $A$ and $C$ are independent, and $P(B \mid C)=0.1$, find $P(A \cup B \cup C)$.

4. In a dice game, the dealer rolls 2 dice, the player rolls 2 dice, and the player wins if their total is larger than the dealer’s. What is the probability that the player wins?

5. A student is taking a multiple-choice test in which each question has four possible answers. She knows the answers to 50% of the questions, can narrow the choices down to two 30% of the time, and does not know anything about 20% of the questions. What is the probability that she will correctly answer a question chosen at random from the test?

6. Binary digits (0’s and 1’s) are sent down a noisy communication channel. They are received as sent with probability 0.9 and an error occurs with probability 0.1. Assuming that 0’s and 1’s are equally likely, what is the probability that a 1 was sent given that a 1 was received?

7. Five pennies are sitting on a table. Four are normal pennies but one has two heads. You pick a coin and flip it four times. You get heads each time. What is the probability you picked up the two-headed penny?

8. Suppose we draw two tickets from a hat that contains tickets numbered 1, 2, 3, and 4. Let $X$ be the first number drawn and $Y$ the second. Find the joint distribution of $X$ and $Y$. Are they independent?
9. Compute $P(X = 2 \mid Y = 3)$ and $P(Y = 3 \mid X = 3)$ for the following joint distribution:

<table>
<thead>
<tr>
<th>$Y \backslash X$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
<td>0.15</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.05</td>
<td>0</td>
<td>0.05</td>
</tr>
</tbody>
</table>

10. A red urn contains 2 red marbles and 3 blue marbles. A blue urn contains 1 red marble and 4 blue marbles. A marble is selected from an urn, the marble is returned to the urn from which it was drawn, and the next marble is drawn from the urn with the color that was drawn. Write the transition matrix for this chain. If the first marble is chosen from the red urn, what is the probability that the third one will be drawn from the blue urn?

11. At Llenroc College, 63% of freshmen who are premed switch to a liberal arts major, while 18% of liberal arts majors switch to premed. In the incoming freshman class is 60% premed and 40% liberal arts majors, what fraction graduate as premed?

12. Why is it true that if all the rows of $p^n$ converge to some vector $\pi$, that $\pi$ gives the long term behavior of the system?

13. Three of every four trucks on the road are followed by a car, while only one out of every five cars is followed by a truck. What fraction of vehicles on the road are trucks?

14. A certain town never has two sunny days in a row. Each day is classified as rainy, cloudy, or sunny. If it is sunny one day then it is equally likely to be cloudy or rainy the next. If it is cloudy or rainy then it remains the same 1/2 of the time, but if it changes it will go to either of the other possibilities with probability 1/4 each. In the long run, what proportion of days in this town are sunny? Cloudy? Rainy?

15. Write the transition matrix for a Markov chain that has a state of period 3, a state of period 2, and no states of period 1.

16. At a manufacturing plant, employees are classified as trainee (R), technician (T) or supervisor (S). Writing Q for an employee who quits, we model their progress through the ranks as a Markov chain with the transition probability

\[
\begin{array}{cccc}
R & T & S & Q \\
R & 0.2 & 0.6 & 0 & 0.2 \\
T & 0 & 0.55 & 0.15 & 0.3 \\
S & 0 & 0 & 1 & 0 \\
Q & 0 & 0 & 0 & 1 \\
\end{array}
\]

where each time step is 2 months. What fraction of recruits eventually make it to supervisor? What is the expected time until a trainee quits or becomes a supervisor?