61. We begin by drawing the upper-left part of the table.

<table>
<thead>
<tr>
<th>$N_1 \backslash N_6$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>$(\frac{1}{6})^2$</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{1}{6})$</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{5}{6})(\frac{1}{6})$</td>
</tr>
<tr>
<td>2</td>
<td>$(\frac{1}{6})^2$</td>
<td>0</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{1}{6})^2$</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{5}{6})(\frac{1}{6})(\frac{1}{6})$</td>
</tr>
<tr>
<td>3</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{1}{6})$</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{1}{6})^2$</td>
<td>0</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{5}{6})(\frac{1}{6})^2$</td>
</tr>
<tr>
<td>4</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{1}{6})^2$</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{5}{6})(\frac{1}{6})(\frac{1}{6})$</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{5}{6})(\frac{1}{6})^2$</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{1}{6})^3$</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{5}{6})(\frac{1}{6})(\frac{1}{6})$</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{5}{6})(\frac{1}{6})(\frac{1}{6})^2$</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{5}{6})(\frac{1}{6})^2$</td>
</tr>
<tr>
<td>6</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{1}{6})^4$</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{5}{6})(\frac{1}{6})(\frac{1}{6})$</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{5}{6})(\frac{1}{6})(\frac{1}{6})^2$</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{5}{6})(\frac{1}{6})^2$</td>
</tr>
</tbody>
</table>

We see a pattern emerging, so we can write our distribution:

$$P(N_1 = i, N_6 = j) = \begin{cases} 0 & 1 \leq i = j, \\ \left(\frac{1}{6}\right)^{i-1}(\frac{5}{6})^{j-1}(\frac{1}{6})^2 & 1 \leq i < j, \\ \left(\frac{1}{6}\right)^{j-1}(\frac{5}{6})^{i-1}(\frac{1}{6})^2 & 1 \leq j < i. \end{cases}$$

64.

$$P(Y = 2 \mid X = 0) = \frac{1}{4}$$

$$P(Y = 2, X = 0) = \frac{1}{4}$$

$$P(Y = 2, X = 0) = \frac{1}{4}$$

$$P(Y = 2, X = 0) + P(Y = 1, X = 0) = 1/4$$

$$0.1 + P(Y = 1, X = 0) = 1/4$$

$$P(Y = 1, X = 0) = 0.3$$
Since $X$ and $Y$ are independent, we know $P(Y = 2, X = 0) = P(Y = 2)P(X = 0)$. So, $0.1 = (0.1 + 0.3)P(Y = 2)$, which means $P(Y = 2) = 0.25$. This forces $P(Y = 2, X = 6)$ to be $0.25 - 0.1 - 0.05 = 0.1$.

Again by independence, $P(Y = 2, X = 3) = P(Y = 2)P(X = 3)$. So, $0.05 = 0.25P(X = 3)$, which means $P(X = 3) = 0.2$. This forces $P(Y = 1, X = 3)$ to be $0.25 - 0.1 - 0.05 = 0.1$.

Since $P(Y = 1) + P(Y = 2) = 1$, we have $P(Y = 1) = 1 - 0.25 = 0.75$. From the rest of the table, $P(Y = 1, X = 6) = 0.75 - 0.3 - 0.15 = 0.3$.

<table>
<thead>
<tr>
<th>$Y \setminus X$</th>
<th>0</th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3</td>
<td>0.15</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.05</td>
<td>0.1</td>
</tr>
</tbody>
</table>

1. (a) $x = 1 - 0.5 - 0.1 = 0.4$
   $y = 1 - 0.2 - 0.4 = 0.4$
   $z = 1 - 0.3 - 0.1 = 0.6$

   (b) $x = 1 - 0.1 - 0.7 = 0.2$
   $y = 1 - 0.2 - 0.3 - 0.5$
   $z = 1 - 0.6 - 0.2 = 0.2$

4. Let the first state be HH, the second be HT, the third be TH, and the fourth be TT.

   (a) $\begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \end{bmatrix}$

   (b) $\begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix}$

5. (a) $\begin{bmatrix} 0.75 & 0.5 & 0.5 \\ 0.75 & 0 & 0.25 \\ 0.75 & 0.25 & 0 \end{bmatrix}$

   (b) $p^2 = \begin{bmatrix} 0.75 & 0.125 & 0.125 \\ 0.1875 & 0.4375 & 0.375 \\ 0.1875 & 0.375 & 0.4375 \end{bmatrix}$, $p^3 = \begin{bmatrix} 0.1875 & 0.40625 & 0.40625 \\ 0.609375 & 0.1875 & 0.203125 \\ 0.609375 & 0.203125 & 0.1875 \end{bmatrix}$

So, the probability the driver is at the airport at time 2 is 0.75, the probability he’s at hotel B at time 2 is 0.125, and the probability he’s at hotel C at time 2 is 0.125. The probability he’s at hotel B at time 3 is 0.40625.
8. \((1 - 0.08)(0.564) + (0.26)(1 - 0.564) = 0.63224\). So, we expect 63.224% of people to have cable in 1995 based on the data in 1990.

\((1 - 0.08)(0.63224) + (0.26)(1 - 0.63224) = 0.6772784\). So, we expect 67.72784% of people to have cable in 2000 based on the data in 1990.

\((1 - 0.08)(0.634) + (0.26)(1 - 0.634) = 0.667844\). So, we expect 67.844% of people to have cable in 2000 based on the data in 1995.

This seems to be a fairly good model, as it gives values close to the observed values.

---

**Additional problem.**

(a) \[
\begin{array}{c|cccc}
X \backslash Y & 0 & 1 & 2 & 3 \\
\hline
1 & \frac{(0)(1)}{6(\omega)} & \frac{(1)(0)}{6(\omega)} & 0 & 0 \\
2 & \frac{(0)(2)}{6(\omega)} & \frac{(1)(1)}{6(\omega)} & \frac{(2)(0)}{6(\omega)} & 0 \\
3 & \frac{(0)(3)}{6(\omega)} & \frac{(1)(2)}{6(\omega)} & \frac{(2)(1)}{6(\omega)} & \frac{(3)(0)}{6(\omega)} \\
4 & 0 & \frac{(1)(3)}{6(\omega)} & \frac{(2)(2)}{6(\omega)} & \frac{(3)(1)}{6(\omega)} \\
5 & 0 & 0 & \frac{(2)(3)}{6(\omega)} & \frac{(3)(2)}{6(\omega)} \\
6 & 0 & 0 & 0 & \frac{1}{6} \\
\end{array}
\]

(b) \(P(X = i) = \frac{1}{6}\) for 1 \(\leq i \leq 6\).

\(P(Y = 0) = \frac{1}{8}\)
\(P(Y = 1) = \frac{7}{24}\)
\(P(Y = 2) = \frac{7}{24}\)
\(P(Y = 3) = \frac{7}{24}\)

(c) \[
\begin{array}{c|cccccc}
x & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
P(X = x \mid Y = 2) & 0 & \frac{3}{35} & \frac{9}{35} & \frac{12}{35} & \frac{2}{7} & 0 \\
\end{array}
\]

(d) \(E(X = x \mid Y = 2) = 1(0) + 2 \left(\frac{4}{35}\right) + 3 \left(\frac{9}{35}\right) + 4 \left(\frac{12}{35}\right) + 5 \left(\frac{2}{7}\right) + 6(0) = \frac{19}{5}\)