16. \( \frac{1}{10} \frac{6}{9} + \frac{1}{10} \frac{5}{9} + \frac{1}{10} \frac{4}{9} + \frac{1}{10} \frac{3}{9} + \frac{1}{10} \frac{3}{9} + \frac{1}{10} \frac{4}{9} + \frac{1}{10} \frac{5}{9} + \frac{1}{10} \frac{6}{9} = \frac{7}{15} \)

(see picture below)
17. Suppose we put \( n \) balls into one of the urns for some fixed \( n \). We can assume without loss of generality that \( 0 \leq n \leq 5 \). Suppose we put \( W \) of the white balls into that urn. Then the probability of drawing a white ball is 
\[
\frac{1}{2} \left( \frac{W}{n} \right) + \frac{1}{2} \left( \frac{5-W}{10-n} \right) = \frac{(10-2n)W+5n}{2n(10-n)}.
\]
Since \( 10-2n \) is always nonnegative, this function is increasing in \( W \). The largest \( W \) can be is \( n \), so the probability is largest when \( W = n \), that is, when all \( n \) of the balls we put into the urn are white.

If we put \( n \) white balls into one of the urns and all the rest of the balls into the other urn, then our probability of choosing a white ball is \( \frac{1}{2} \) if \( n = 0 \) and \( \frac{1}{2} + \frac{1}{2} \left( \frac{5-n}{10-n} \right) \) if \( 0 < n \leq 5 \). The second probability is always at least as large as the first, so we can rule out \( n = 0 \). This probability is largest when \( n = 1 \).

So, our probability of drawing a white ball is maximized when we place 1 white ball in one urn and all the other balls in the second urn.

24. 
\[
P(\text{late}) = P(\text{subway})P(\text{late} \mid \text{subway}) + P(\text{bus})P(\text{late} \mid \text{bus})
= 0.7(0.2) + 0.3(0.4)
= 0.26
\]

32. Let \( p_1 \) be the probability that the first boy gets the first basket, \( p_2 \) be the probability that the second boy gets the first basket, and \( p_3 \) be the probability that the third boy gets the first basket.

\[
p_1 = 0.2 + (0.8)(0.7)(0.5)p_1
\]
\[
p_1 = 0.2778
\]
\[
p_2 = (0.8)(0.3) + (0.8)(0.7)(0.5)p_2
\]
\[
p_2 = 0.3333
\]
\[
p_3 = 1 - p_1 - p_2
\]
\[
p_3 = 0.3889
\]

39. Let \( C_B \) be the event the cab was blue, \( C_G \) be the event the cab was green, and \( W_B \) be the event the witness said the cab was blue.
\[ P(C_B \mid W_B) = \frac{P(C_B \cap W_B)}{P(W_B)} = \frac{P(C_B)P(W_B \mid C_B)}{P(W_B \mid C_B) + P(C_G)P(W_B \mid C_G)} = \frac{(0.15)(0.8)}{(0.15)(0.8) + (0.85)(0.2)} = \frac{0.12}{0.12 + 0.17} = 0.41 \]

40. Let \( S = \) be the event the day was snowy, \( NS \) be the event the day was not snowy, and \( C \) be the event the student was in class.

\[ P(S \mid C) = \frac{P(S \cap C)}{P(C)} = \frac{P(S)P(C \mid S)}{P(S)P(C \mid S) + P(NS)P(C \mid NS)} = \frac{(0.2)(0.4)}{(0.2)(0.4) + (0.8)(0.7)} = \frac{0.08}{0.08 + 0.56} = 0.125 \]

55. Let \( C \) be the event she is a Conservative, \( I \) be the event she is an Independent, \( L \) be the event she is a Liberal, and \( V \) be the event she is a voter.

\[ P(L \mid V) = \frac{P(L \cap V)}{P(V)} = \frac{P(L)P(V \mid L)}{P(L)P(V \mid L) + P(I)P(V \mid I) + P(C)P(V \mid C)} = \frac{(0.5)(0.8)}{(0.5)(0.8) + (0.2)(0.5) + (0.3)(2/3)} = \frac{0.4}{0.4 + 0.1 + 0.2} = 0.5714 \]
61. We begin by drawing the upper-left part of the table.

<table>
<thead>
<tr>
<th>$N_1 \setminus N_6$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>$(\frac{1}{6})^2$</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{1}{6})$</td>
<td>$(\frac{1}{6})(\frac{5}{6})^2(\frac{1}{6})$</td>
</tr>
<tr>
<td>2</td>
<td>$(\frac{1}{6})^2$</td>
<td>0</td>
<td>$(\frac{1}{6})(\frac{5}{6})^2$</td>
<td>$(\frac{1}{6})(\frac{5}{6})^2(\frac{1}{6})$</td>
</tr>
<tr>
<td>3</td>
<td>$(\frac{1}{6})(\frac{5}{6})(\frac{1}{6})$</td>
<td>$(\frac{1}{6})(\frac{5}{6})^2$</td>
<td>0</td>
<td>$(\frac{1}{6})^2(\frac{1}{6})^2$</td>
</tr>
<tr>
<td>4</td>
<td>$(\frac{1}{6})(\frac{5}{6})^2(\frac{1}{6})$</td>
<td>$(\frac{1}{6})(\frac{5}{6})^2(\frac{1}{6})$</td>
<td>0</td>
<td>$(\frac{1}{6})^3(\frac{1}{6})^2$</td>
</tr>
<tr>
<td>5</td>
<td>$(\frac{1}{6})^2(\frac{1}{6})$</td>
<td>$(\frac{1}{6})(\frac{5}{6})^3(\frac{1}{6})$</td>
<td>$(\frac{1}{6})(\frac{5}{6})^2(\frac{1}{6})$</td>
<td>$(\frac{1}{6})^3(\frac{1}{6})^2$</td>
</tr>
<tr>
<td>6</td>
<td>$(\frac{1}{6})(\frac{5}{6})^3(\frac{1}{6})$</td>
<td>$(\frac{1}{6})(\frac{5}{6})^3(\frac{1}{6})$</td>
<td>$(\frac{1}{6})(\frac{5}{6})^2(\frac{1}{6})$</td>
<td>$(\frac{1}{6})^3(\frac{1}{6})(\frac{1}{6})$</td>
</tr>
</tbody>
</table>

We see a pattern emerging, so we can write our distribution:

$$P(N_1 = i, N_6 = j) = \begin{cases} 0 & 1 \leq i = j, \\ (\frac{1}{6})^{i-j}(\frac{5}{6})^{j-i-1}(\frac{1}{6})^2 & 1 \leq i < j, \\ (\frac{1}{6})^{j-i}(\frac{5}{6})^{i-j-1}(\frac{1}{6})^2 & 1 \leq j < i. \end{cases}$$

64.

$$P(Y = 2 \mid X = 0) = 1/4$$

$$P(Y = 2, X = 0)$$

$$P(Y = 2, X = 0) = 1/4$$

$$P(Y = 2, X = 0) + P(Y = 1, X = 0)$$

$$0.1$$

$$P(Y = 1, X = 0) = 0.3$$

Since $X$ and $Y$ are independent, we know $P(Y = 2, X = 0) = P(Y = 2)P(X = 0)$. So, $0.1 = (0.1 + 0.3)P(Y = 2)$, which means $P(Y = 2) = 0.25$. This forces $P(Y = 2, X = 6)$ to be $0.25 - 0.1 - 0.05 = 0.1$.

Again by independence, $P(Y = 2, X = 3) = P(Y = 2)P(X = 3)$. So, $0.05 = 0.25P(X = 3)$, which means $P(X = 3) = 0.2$. This forces $P(Y = 1, X = 3)$ to be $0.2 - 0.05 - 0.15$.

Since $P(Y = 1) + P(Y = 2) = 1$, we have $P(Y = 1) = 1 - 0.25 = 0.75$. From the rest of the table, $P(Y = 1, X = 6) = 0.75 - 0.3 - 0.15 = 0.3$.

<table>
<thead>
<tr>
<th>$Y \setminus X$</th>
<th>0</th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3</td>
<td>0.15</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.05</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Additional problem. (a) \[
\begin{array}{c|cccc}
X \setminus Y & 0 & 1 & 2 & 3 \\
\hline
1 & \frac{6(3)}{6(3)} & \frac{(1)(6)}{6(3)} & 0 & 0 \\
2 & \frac{6(3)}{6(3)} & \frac{(1)(6)}{6(3)} & \frac{(2)(6)}{6(3)} & 0 \\
3 & \frac{6(3)}{6(3)} & \frac{(1)(6)}{6(3)} & \frac{(2)(6)}{6(3)} & \frac{(3)(6)}{6(3)} \\
4 & 0 & \frac{(1)(6)}{6(3)} & \frac{(2)(6)}{6(3)} & \frac{(3)(6)}{6(3)} \\
5 & 0 & 0 & \frac{(1)(6)}{6(3)} & \frac{(2)(6)}{6(3)} \\
6 & 0 & 0 & 0 & \frac{(1)(6)}{6(3)} \\
\end{array}
\]

(b) \[P(X = i) = \frac{1}{6} \text{ for } 1 \leq i \leq 6.\]
\[P(Y = 0) = \frac{1}{8},\]
\[P(Y = 1) = \frac{7}{24},\]
\[P(Y = 2) = \frac{7}{24},\]
\[P(Y = 3) = \frac{7}{24} \]

(c) \[
\begin{array}{c|cccccc}
x & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
P(X = x \mid Y = 2) & 0 & \frac{4}{35} & \frac{9}{35} & \frac{12}{35} & \frac{7}{35} & 0 \\
\end{array}
\]

(d) \[E(X = x \mid Y = 2) = 1(0) + 2\left(\frac{4}{35}\right) + 3\left(\frac{9}{35}\right) + 4\left(\frac{12}{35}\right) + 5\left(\frac{7}{35}\right) + 6(0) = \frac{19}{5} \]