Motivation

One of the key features of the theory of plabic networks developed by Postnikov is the fact that if a plabic graph is reduced, the weight can be uniquely recovered from boundary measurements. On surfaces more complicated than a disk this property is lost. We study a certain semi-local transformation of weights for plabic networks on a cylinder that preserve boundary measurements. We call this a plabic R-matrix. Plabic R-matrices have underlying cluster algebra structure, generalizing recent work of Inoue-Lam-Pylyavskyy. Special cases of transformations we consider include geometric R-matrices appearing in Berenstein-Kazhdan theory of geometric crystals, and also certain transformations appearing in a recent work of Goncharov-Shen.

Boundary Measurements

Given a planar directed network on a cylinder with positive real edge weights \( \{x_e\} \) and boundary vertices \( b_1, \ldots, b_n \), we can define the boundary measurements

\[
M_Q = \sum_{\text{paths } P} (-1)^{\text{ind}(C_P)} \chi_{\text{init}(P)} \prod_{e \in P} x_e.
\]

Inverse Boundary Problem: What information about a planar directed network can be recovered given the boundary measurements?

Plabic Networks

We can redraw our graph so every vertex either has exactly one incoming edge or one outgoing edge. We will denote the former by a white vertex and the latter by a black vertex.

Face and Trail Weights

We can rewrite our network in terms of face and trail weights without changing the boundary measurements.

Plabic R-Matrices

Plabic R-matrices have the following properties:

- They preserve the boundary measurements.
- They are involutions.
- They give the only choices of weights on a fixed cylindric 2-loop plabic graph that preserve the boundary measurements.
- They satisfy the braid relation.

References


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