A more honest calculation of an AR-quiver (in rep'n finite case)

1. Compute the project and injectives.

2. Compute $rad\, P_i \rightarrow P_i$ irreducibly.

3. Apply $T^{-1}$ to each and continue until you reach indecomposables.

General $\circ T$ applied to indecomposables gives an indecomposable.

$$\begin{array}{c}
\begin{align*}
1_i & \rightarrow 3_i & 1 & \rightarrow 3 & 3 & \rightarrow 3 & 4
\end{align*}
\end{array}$$

$$\begin{array}{c}
\begin{align*}
P_1 & = 1 & P_2 & = 2 & P_3 & = 2^{3} & P_4 & = 4
\end{align*}
\end{array}$$

$$\begin{array}{c}
\begin{align*}
& \text{minimal injective presentation}
\end{align*}
\end{array}$$

3. $4 \rightarrow 3 \rightarrow 0$

4. $\text{coker}(f) \rightarrow 1 \oplus 3 \rightarrow 0$

5. $\text{since } T^{-1}(2) \text{ must be indecomposable}$

A large class of non-hereditary tame algebras for which we can completely understand every indecomposable.

Many tame algebras degenerate to string algebras (tame local algebras and biserial algebras).
12. (25 points) A lamina (i.e. thin plate) enclosed by \( y = x \) and \( y = x^2 \) has density \( \rho(x, y) = x \).

Find its center of mass using integration.

Ex: \( 1k \) \([\begin{array}{c}
\frac{1}{2} \\
4
\end{array}] \) (pilhs of length 3)

\[ A = 1k \frac{A}{x^2} \]

is a string algebra if

i) \( I \) is generated by monomials of arrows

ii) each vertex \( i \in Q \) has at 2 incoming arrows and 2 outgoing arrows

iii) for each \( \beta \in I \) there is at most one \( \alpha \in Q \) s.t. \( \alpha \beta \in I \) and

iv) at most one arrow \( \gamma \) s.t. \( \beta \gamma \in I \).

Then [Wald-Varchesich]: The indecomposable \( A \)-modules are exactly the string and band modules.

A string in \( Q \) is a sequence \( w = w_1 \cdots w_k \) of arrows of \( Q \) where

- \( w \) has no redundant substrings,
- no consecutive substring that lies in \( I \).

A band is a string that starts and ends at the same vertex but is not a proper power of another string.

- \( M(w) = (v_i)_{i \in I} \) \( (v_i)_0 \)
- \( V_i \)

The action of \( v \alpha \) is induced by the relevant identity maps.

\[ M(w) = (v_i)_{i \in I} \]

\( (v_i)_{0} \)

\( V_i \)

\( \alpha \)

\[ \sum_{x \in X} \alpha_x \]

\[ M(1k) = 1k \]

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Example: \( w = 3, 3, 1, 1, 1, 1, \lambda \) 

For each \( \alpha \in \Omega \), there is a unique string \( \alpha x = x_l \circ \cdots \circ x_r \) where \( x_l, x_r \) consist of formal inverses of arrows in \( M \) starting in a peak and end in a peak.

String terminology:
- \( w \) starts on a peak if for every \( i \), \( \alpha_i w \) is a string.
- \( w \) ends in a deep if for every \( i \), \( \alpha_i w \) is a string.
- \( w \) starts in a deep if for every \( i \), \( \alpha_i w \) is a string.
- \( w \) ends on a peak if for every \( i \), \( \alpha_i w \) is a string.

Theorem: A string algebra. Assume \( M(w) \) is not injective. The almost split sequence is not injective. The almost split sequence starting in \( M(w) \) is not injective.

(2) If \( w \) does not start but ends on a peak, it is

\[ M(l(w)) \oplus M(w_{<}) \rightarrow M(w_{=}) \rightarrow 0 \]

Example:

\[ w = 3, 1, 1, 1, 1, 1, \lambda \]