Indecomposable modules: A Λ-module M is <u>indecomposable</u> if $M \neq 0$ and $M = U \oplus V$ implies $U = 0$ or $V = 0$.

There can be ∞'ly many!

Thm: M is indecomposable iff the fda $\operatorname{End}_\Lambda(M)$ is a local ring. (unique maximal left ideal = unique maximal right ideal)

Krull-Schmidt: For any Λ-module M, if

$$M \cong U_1 \oplus \cdots \oplus U_r, \ M \cong V_1 \oplus \cdots \oplus V_s$$

with U_i, V_j indec., then $r = s$ and after reordering $U_i \cong V_i$.

Thm: M - a Λ-module. $\exists$ expressions $M = U \oplus V$ biject with expressions $1 = e + f$ in $\operatorname{End}_\Lambda(M)$ where e, f are idempotents s.t. $ef = fe = 0$ (e, f are orthogonal idempotents)

Cor: Expressions $M = U_1 \oplus \cdots \oplus U_r$ biject with expressions $1 = e_1 + \cdots + e_r$ in $\operatorname{End}_\Lambda(M)$ where e_i's are orthogonal idempotents. If U_i's are indec., e_i's are primitive, i.e. e_i can not be written as a non trivial sum of orthogonal idempotents.

$U_i = e_i(M)$

Fact: Every indec. projective is isomorphic to a summand of ${}_\Lambda\Lambda$, so can be realized as Λe_i for a primitive orthogonal idempotent e of $\Lambda^{op} \cong \operatorname{End}_\Lambda({}_\Lambda\Lambda)$