1| A more honest calculation of an AR-quiver (in rep'n finite case)

① Compute the ~~project~~ P_is and I_is ~~② Exercise~~

$$\begin{tikzcd} 1 \arrow[dr] & & 3 \arrow[dl] \arrow[dr] & \\ & 2 & & 4 \end{tikzcd}$$

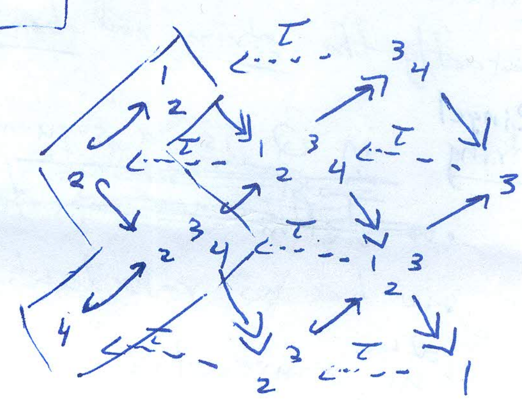
② Compute ~~their~~ radicals ($\operatorname{rad} P_i \hookrightarrow P_i$ irreducible)

③ Apply τ^{-1} $= \mathrm{Hom}_A(-,A) \circ D$, $D = \mathrm{Hom}_k(-,k)$, to each and continue until you reach injectives.

General ~~facts~~: • If Λ, rep'n finite and Q connected, $\Gamma(A\text{mod})$ has a single component.
• τ, τ^{-1} applied to indecomposables gives an indecomposable

① P_is: $P_1 = \begin{smallmatrix}1\\2\end{smallmatrix}$, $P_2 = 2$, $P_3 = \begin{smallmatrix}&3&\\2&&4\end{smallmatrix}$, $P_4 = 4$

I_is: $I_1 = 1$, $I_2 = \begin{smallmatrix}1&&3\\&2&\end{smallmatrix}$, $I_3 = 3$, $I_4 = \begin{smallmatrix}3\\4\end{smallmatrix}$ $\qquad \nu := D\mathrm{Hom}_A(-,A)$



② $\operatorname{rad} P_1 = P_2$, $\operatorname{rad} P_3 = P_2 \oplus P_4$

③ $4 \longrightarrow \underset{I_4}{\begin{smallmatrix}3\\4\end{smallmatrix}} \longrightarrow \underset{I_3}{3} \longrightarrow 0$

minimal injective presentation

$\downarrow \nu^{-1}$ $\qquad \downarrow \nu^{-1} := \mathrm{Hom}_A(D(_AA), -)$ inverse of Nakayama functor

$\underset{P_4}{4} \xrightarrow{f} \underset{P_3}{\begin{smallmatrix}&3&\\2&&4\end{smallmatrix}} \longrightarrow \operatorname{coker}(f) = \tau^{-1}(4) = \begin{smallmatrix}&3\\2&\end{smallmatrix}$

$2 \longrightarrow \underset{I_2}{\begin{smallmatrix}1&&3\\&2&\end{smallmatrix}} \longrightarrow \underset{I_1 \oplus I_3}{1 \oplus 3} \longrightarrow 0$

$\downarrow \qquad \downarrow$

$\underset{P_2}{2} \xrightarrow{f} \begin{smallmatrix}1\\2\end{smallmatrix} \oplus \begin{smallmatrix}&3&\\2&&4\end{smallmatrix} \longrightarrow \operatorname{coker}(f) = \begin{smallmatrix}1&&3&\\&2&&4\end{smallmatrix}$ (since $\tau^{-1}(2)$ must be indecomposable

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String Algebras: ~~There is~~ A large class of non-hereditary tame algebras for which one can concretely ~~understand~~ understand every indecomposable.

Many tame algebras degenerate to string algebras.
(tame local algebras and biserial algebras

2

12. (25 points) A lamina (i.e. thin plate) enclosed by $y = x$ and $y = x^2$ has density $\rho(x, y) = x$.

Find its center of mass **using integration**.

$\Bbbk = \bar{\Bbbk}$

fin. dim'l

Ex: $\Bbbk\begin{pmatrix} 2 & \longrightarrow & 3 \\ \uparrow & & \downarrow \\ 1 & \longleftarrow & 4 \end{pmatrix} / \langle \text{paths of length } 3 \rangle$

$A = \Bbbk Q / I$ is a string algebra if

i) I is generated by monomials of arrows

ii) each vertex $i \in Q_0$ has at 2 incoming arrows and 2 outgoing arrows

iii) for each $\beta \in Q_1$ $\exists$ at most one $\alpha \in Q_1$ s.t. $\alpha\beta \notin I$ and $\exists$ at most one arrow γ s.t. $\beta\gamma \in I$.

Thm [Wald-Waschbüsch]: The indecomposable A-modules ~~of~~ a string algebra A are exactly the string and band modules.

(Butler-Ringel)

A string in Q is a sequence $w = w_1 \cdots w_k$ of arrows and formal inverses of arrows of Q where $x_1 \overset{w_1}{\longleftrightarrow} x_2 \cdots x_k \overset{w_k}{\longleftrightarrow} x_{k+1}$, x_i is a vertex of w_i and w_{i+1}

$w_i = \alpha_i$ if $x_i \to x_{i+1}$

$w_i = \alpha_i^{-1}$ if $x_i \leftarrow x_{i+1}$

- ~~w defines a connected subquiver of Q~~
- w has no redundant substrings, and
- w has no consecutive substring that lies in I.

A band is a string that starts and ends at the same vertex but is not a proper power of another string.

$M(w) = ((V_i)_i, (\varphi_\alpha)_\alpha)$ $\quad V_i = \Bbbk^{\#\{x_j \text{ visits } x_i\}}$

string module

The action of φ_α is induced by the relevant identity maps

assign identity ~~morphisms~~ maps where a nonzero maps can be assigned

band module

$M(w, m, \phi) = ((V_i)_i, (\varphi_\alpha)_\alpha)$

$\phi \in \mathrm{Aut}(\Bbbk^m)$

$V_i = \begin{cases} \Bbbk^n : i = x_j \text{ some } j \neq k+1 \\ 0 : \text{o.w.} \end{cases}$ $\quad \varphi_\alpha = \begin{cases} 1 : \alpha = w_i \text{ some } i \neq k \\ \phi : \alpha = w_k \\ 0 : \text{o.w.} \end{cases}$

The action of φ_α is induced by the relevant identity maps when $\alpha = w_i$ for $i \neq k$ and by ϕ when $\alpha = w_k$.

Ex: $Q = 1 \underset{\alpha_2}{\overset{\alpha_1}{\leftleftarrows}} 2$

$M(1 \xleftarrow{\alpha_1} 2 \xrightarrow{\alpha_2}, 1, \lambda) = \Bbbk \underset{\lambda}{\leftleftarrows} \Bbbk = V_{\lambda,1}$ GOOD

Fact: $M(w) \cong M(w^{-1})$

Ex: $\Bbbk\left(\begin{tikzcd} 1 \arrow[r, "\delta"] \arrow[d, "\eta"'] & 2 \arrow[r, "\gamma"] & 4 \arrow[d, "\alpha"] \\ & 3 \arrow[ur, "\beta"] & 5 \arrow[l, "\theta"] \end{tikzcd}\right) \Big/ \langle \alpha\beta, \beta\theta, \theta\alpha \rangle$ (Jacobian algebra of type $\tilde{A}_4$)

__String__ $w = \begin{tikzcd} 3 \arrow[d, "w_1"] & & 1 \arrow[dl, "w_3"'] \arrow[dr, "w_4"] & & \\ 4 & 2 \arrow[l, "w_2"] & & 3 \arrow[d, "w_5"] & \\ & & & 4 & \end{tikzcd}$ $\rightsquigarrow$ $M(w) = \begin{tikzcd} & \Bbbk \arrow[dr, "{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]}"] & \\ \Bbbk \arrow[ur, "1"] \arrow[dr, "{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]}"'] & & \Bbbk^2 \arrow[d, "0"] \\ & \Bbbk^2 \arrow[ur, "{\left[\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right]}"] & 0 \arrow[l, "0"] \end{tikzcd}$ $\left(M(w) \cong M(w^{-1})\right)$

__string module__

__band__ $b = \begin{tikzcd} & 2 \arrow[dr, "w_2"] & \\ 1 \arrow[ur, "w_1"] \arrow[dr, "w_4"'] & & 4 \\ & 3 \arrow[ur, "w_3"'] & \end{tikzcd}$ $\rightsquigarrow M(b, m, \lambda) = \begin{tikzcd} & \Bbbk^m \arrow[dr, "I_m"] & \\ \Bbbk^m \arrow[ur, "I_m"] \arrow[dr, "\lambda"'] & & \Bbbk^m \arrow[d, "0"] \\ & \Bbbk^m \arrow[ur, "I_m"] & 0 \arrow[l, "0"] \end{tikzcd}$

$m \in \mathbb{N}$, $\lambda \in \operatorname{Aut}(\Bbbk^m)$

__band modules__

$\left(M(b, m, \lambda) \cong M(b^{-1}, m, \lambda) \cong M(b', m, \lambda)\right)$

b' a cyclic permutation of b.

Ex: $Q = 1 \underset{\alpha_2}{\overset{\alpha_1}{\leftleftarrows}} 2$ $\quad M(1 \xleftarrow{\alpha_1} 2 \xrightarrow{\alpha_2} 1, 1, \lambda) = \Bbbk \underset{\lambda}{\overset{1}{\leftleftarrows}} \Bbbk = V_{\lambda,1}$

__String terminology__ $w = x_1 \overset{w_1}{\longleftrightarrow} x_2 \cdots x_k \overset{w_k}{\longleftrightarrow} x_{k+1}$ $\quad w_i = \alpha_i$ if $x_i \to x_{i+1}$, α_i^{-1} if $x_i \leftarrow x_{i+1}$

w starts on a __peak__ if $\nexists\, \alpha \in Q$, s.t. αw is a string

ends — " ———— $w\alpha^{-1}$ — " —

start in a __deep__ if ~~$\nexists$~~ — " — $\alpha^{-1} w$ — " —

ends in ~~a~~ __deep__ — " — $w\alpha$ — " —.

__Thm__ [Butler–Ringel] 1987 $A = \Bbbk Q/I$ a string algebra. ~~Assume~~ $M(w)$ ~~is~~ not injective. The almost split sequence starting in $M(w)$ ~~is~~ is as follows.

(2) ~~If~~ w does not start but ends on a peak, it is

$$0 \to M(w) \to M({}_h w) \oplus M(w_c) \to M({}_h w_c) \to 0$$

V_α, U_α

w_c: remove ~~right~~ segment of inverses starting at right end and the arrow preceding this segment. $(\beta\gamma^{-1}\delta^{-1})$

__Example:__ $w = 3 \xrightarrow{\beta} 4 \xleftarrow{\gamma} 2 \xleftarrow{\delta} 1$ $\quad w_c = 3$

${}_h w = 5 \leftarrow 4 \leftarrow 3 \leftarrow 1 \xrightarrow{\eta} 3 \to 4 \leftarrow 2 \leftarrow 1$

add the right V_η so the result starts in a deep.

For each $\alpha \in Q_1$, there is a unique string $M_\alpha = V_\alpha \alpha U_\alpha$ where V_α, U_α consist of formal inverses of arrows and M_α starts in a deep and ends in a peak.