3rd midterm for MATH 1272: Calculus II, section 030

Name: **Section Number:**
ID #: **Teaching Assistant:**
Instructions:

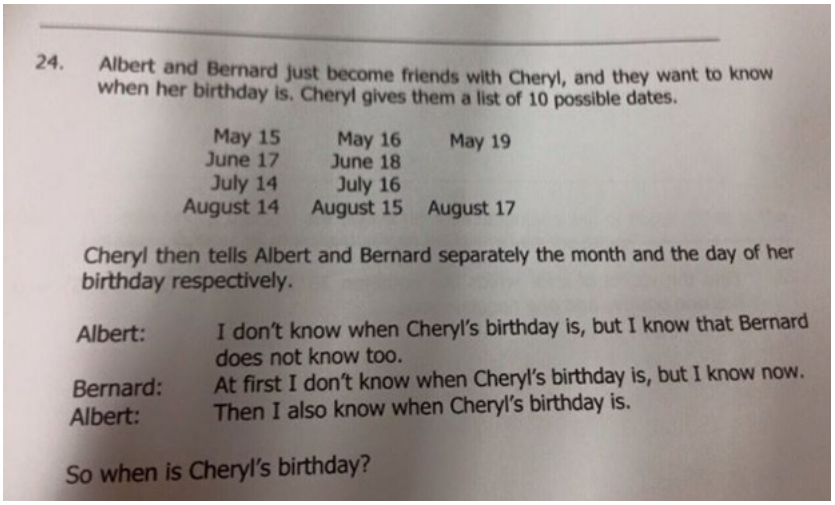
- *Please don't turn over this page until you are directed to begin.*
- There are 5 problems on this exam, and all except problem 2 have multiple parts.
- There are 7 pages to the exam, including this page. All of them are one-sided. If you run out of room on the page that you're working on, use the back of the page.
- Please show all your work. Answers unsupported by an argument will get little credit.
- Scientific calculators are allowed. No books or notes are allowed. Please turn off your cell phones.

Grading summary

Problem:	1	2	3	4	5	total
Possible:	10 points	5 points	30 points	15 points	20 points	80 points
Grade:						

Some helpful formulas

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$	$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$	$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$
$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$	$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$	$\tan^{-1}(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$



24. Albert and Bernard just become friends with Cheryl, and they want to know when her birthday is. Cheryl gives them a list of 10 possible dates.

May 15	May 16	May 19
June 17	June 18	
July 14	July 16	
August 14	August 15	August 17

Cheryl then tells Albert and Bernard separately the month and the day of her birthday respectively.

Albert: I don't know when Cheryl's birthday is, but I know that Bernard does not know too.
Bernard: At first I don't know when Cheryl's birthday is, but I know now.
Albert: Then I also know when Cheryl's birthday is.

So when is Cheryl's birthday?

1. **(10 points total, 5 points each)** Determine whether the following sequences $\{a_n\}$ converge or diverge. If they converge, compute the limit, $\lim_{n\to\infty} a_n$.

   (a) $a_n = \dfrac{2+7n^2}{n+n^2}$.

   (b) $a_n = \dfrac{n^2}{\sqrt{n^2-n}}$.

2. **(5 points)** Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n \cdot 5^n}$. How many terms of the series do we need to sum in order to be within an error of at most 10^{-4} of the actual infinite sum?

3. **(30 points total, 5 points each)** Are the following series absolutely convergent, conditionally convergent, or divergent? Justify for your answer. If they converge, you do *not* need to compute their sum.

(a) $\displaystyle\sum_{n=2}^{\infty}(-1)^n\left(\frac{n^3}{n^4-1}\right)$.

(b) $\displaystyle\sum_{n=1}^{\infty}\ln\left(\frac{n^2+1}{2n^2+1}\right)$.

(c) $\displaystyle\sum_{n=2}^{\infty}\left(\frac{-n}{2n+1}\right)^{5n}$.

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(d) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3}{5^n}+\frac{2}{n}\right)$.

(e) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln n}$.

(f) $\displaystyle\sum_{n=1}^{\infty}\frac{3^n\cdot n^2}{n!}$.

4. **(15 points total, 5 points each)** Find the radius of convergence of the following power series:

(a) $\sum_{n=1}^{\infty}(-4)^n(x-5)^n$.

(b) $\sum_{n=1}^{\infty} n^n x^n$.

(c) $\sum_{n=1}^{\infty} \frac{(x-7)^n}{n!}$.

5. **(20 points total, 5 points each)** Find a power series representation for the following functions and determine the radius of convergence:

   (a) $f(x) = x \cos x$, centered at $a = 0$.

   (b) $f(x) = \cos x$, centered at $a = \pi$.

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(c) $f(x) = \dfrac{x}{9 + x^2}$, centered at $a = 0$.

(d) $f(x) = \ln(5 - x)$, centered at $a = 4$.