Final exam for MATH 1272: Calculus II, Spring 2015

Name: **Section Number:**
ID #: **Teaching Assistant:**
Signature:

General Instructions: *Please don't turn over this page until you are directed to begin.* There are 19 problems on this exam. There are 21 pages to the exam, including this page. Check to see if any are missing. Do your work in the space provided in this booklet. Scientific calculators are allowed. No books or notes are allowed. Please turn off your cell phones.

Instructions for the machine-graded part: (Questions 1-13) You **must** use a soft pencil (No. 1 or No. 2) to answer this part. Do not tear or fold the answer sheet, and carefully enter all the requested information according to the instructions that you receive. Do not make any stray marks on the answer sheet. When you have decided on a correct answer to a given question, circle the answer in the booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. If you give two different answers, the question will be marked wrong. There is no penalty for guessing. If you don't answer a question, skip the corresponding line on the answer sheet and go on to the next question.

Instructions for the hand-graded part: (Questions 14-19) Please show all your work in the space provided below each question. Answers unsupported by an argument will get little credit.

Grading summary

Problem:	$1-13$	14	15	16	17	18	19	**Total**
Possible:	130 points	10 points	15 points	10 points	10 points	10 points	10 points	195 points
Grade:								

Some helpful formulas

$\sin^2(x) + \cos^2(x) = 1$	$\tan^2(x) + 1 = \sec^2(x)$	$1 + \cot^2(x) = \csc^2(x)$
$2\sin^2(x) = 1 - \cos(2x)$	$2\cos^2(x) = 1 + \cos(2x)$	$2\sin(x)\cos(x) = \sin(2x)$
$\int \tan(x)\ dx = \ln\lvert\sec(x)\rvert + C$	$\int \sec(x)\ dx = \ln\lvert\sec(x) + \tan(x)\rvert + C$	$\int \csc(x)\ dx = \ln\lvert\csc(x) - \cot(x)\rvert + C$
$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x)$	$\int \frac{1}{1+x^2} dx = \tan^{-1}(x) + C$	$\int \tan^2(x)\ dx = \tan(x) - x + C$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$	$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$	$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$
$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$	$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$	$\tan^{-1}(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

1. **(10 points)** The indefinite integral $\displaystyle\int \frac{dt}{2t^2+3t+1}$ is equal to:

(a) $\ln|2t^2+3t+1|+C$;

(b) $\ln|2t+1|-\ln|t+1|+C$;

(c) $\ln|4t+3|+C$;

(d) $\dfrac{1}{2t+1}-\dfrac{1}{t+1}+C$;

(e) $2\ln|t+1|-\frac{1}{2}\ln|2t+1|+C$.

2. **(10 points)** The definite integral $\int_1^2 p^2 \ln p \, dp$ is equal to:

(a) $4 \ln 2$;

(b) $\dfrac{4 \ln 2 - 3}{3}$;

(c) $\dfrac{8 \ln 2 - 7}{3}$;

(d) $\dfrac{8 \ln 2 - 3}{3}$;

(e) $\dfrac{8 \ln 2}{3} - \dfrac{7}{9}$.

3. **(10 points)** The indefinite integral $\int \frac{\sqrt{x^2-49}}{x}\, dx$ is equal to:

(a) $\sqrt{x^2-49} - 7\sec^{-1}\left(\frac{x}{7}\right) + C$;

(b) $\frac{x}{2}\sqrt{x^2-49} - \frac{49}{2}\ln|x+\sqrt{x^2-49}| + C$;

(c) $\frac{1}{3}(x^2-49)^{3/2} + C$;

(d) $\sqrt{x^2-49} - 7\cos^{-1}\left(\frac{x}{7}\right) + C$;

(e) None of the above.

4. **(10 points)** The area of the region bounded by the polar curve $r = e^{\theta}$ and the rays $\theta = 0$ and $\theta = 1$ is

(a) $\frac{1}{4}$;

(b) $\frac{1}{4}(e^2 - 1)$;

(c) $\frac{1}{4}e^2$;

(d) $\frac{1}{2}(e - 1)$;

(e) None of the above.

5. **(10 points)** What is the value of the integral:

$$\int_0^1 \frac{1}{2-3x}dx.$$

(a) $-\frac{1}{3}(\ln(-1) - \ln(2))$;

(b) $\frac{1}{3}\ln(2)$;

(c) $-\frac{1}{3}\ln\left(\frac{1}{2}\right)$;

(d) It diverges;

(e) It converges, but the solution cannot be expressed in elementary terms.

6. **(10 points)** What is the distance between the planes

$$x - 2y + 4z = 1 \;\text{ and }\; 2x - 4y + 3z = -14?$$

(a) 0;

(b) 15;

(c) $\sqrt{6}$;

(d) 22;

(e) $\sqrt{22}$.

7. **(10 points)** The solution to the differential equation $xy' = y + x^2 \sin(x)$ with initial value $y(\pi) = 0$ is

(a) $y = \dfrac{\sin(x)}{x} - \cos(x);$

(b) $y = \dfrac{\sin(x)}{x} - \cos(x) - \dfrac{\pi}{x};$

(c) $y = -x\cos(x) - \pi;$

(d) $y = -x\cos(x) - x;$

(e) There is no solution to this differential equation with initial value $y(\pi) = 0$.

8. **(10 points)** What is the length of the polar curve $r = \theta^2$, $0 \le \theta \le 2\pi$?

(a) $\frac{8}{3}\left((\pi^2+1)^{3/2}-1\right)$;

(b) $\frac{8}{3}(\pi^2+1)^{3/2}$;

(c) $\dfrac{8\pi^3}{3}$;

(d) $8\pi^4$;

(e) None of the above.

9. **(10 points)** What is the area enclosed by one loop of the three-leaved rose $r = \sin(3\theta)$?

(a) $\frac{\pi}{12}$;

(b) $\frac{2}{3}$;

(c) 0;

(d) $\frac{\pi}{6}$;

(e) $\frac{1}{3}$.

10. **(10 points)** The infinite sum $\sum_{n=0}^{\infty} \frac{(\ln 5)^n}{n!}$ equals

(a) 0;

(b) $\ln 5$;

(c) 5;

(d) e^5;

(e) ∞.

11. **(10 points)** The following equation represents a sphere. What are its center and radius?

$$x^2 + y^2 + z^2 - 2x - 4y + 8z = 15$$

(a) center $= (1, 2, 4)$, radius $= \sqrt{15}$;

(b) center $= (1, 2, -4)$, radius $= \sqrt{15}$;

(c) center $= (1, 2, 4)$, radius $= 6$;

(d) center $= (1, 2, -4)$, radius $= 6$;

(e) center $= (-1, -2, 4)$, radius $= 6$.

12. **(10 points)** What is the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n}{4^n}(x+1)^n$?

(a) 0;

(b) $\frac{1}{4}$;

(c) 2;

(d) 4;

(e) ∞.

13. **(10 points)** The curve $x = \sqrt[3]{y}$ on the interval $0 \le x \le 1$ is rotated around the x-axis. What is the surface area of the resulting surface?

(a) $\dfrac{3\pi}{2}$;

(b) $\dfrac{\pi}{2}$;

(c) infinite;

(d) $2\pi^{4/3}$;

(e) $\dfrac{\pi}{27}(\sqrt{1000} - 1)$.

14. **(10 points)** A tank with a capacity of 400 L is full of a mixture of water and sugar with the concentration 0.05 g/L. In order to reduce the concentration of sugar, fresh water is pumped into the tank at a rate of 4 L/s. The mixture is kept stirred and is pumped out at a rate of 10 L/s. Find the amount of sugar in the tank as a function of time.

15. **(10 points)** Solve the initial-value problem

$$(1+\cos x)y' = (1+e^{-y})\sin x, \quad y(0)=0.$$

16. **(15 points total, 5 points each)** Determine whether each of the following series converges or diverges. If it is convergent, find its sum. Show your reasoning.

(a) $\sum_{n=1}^{\infty} \frac{1+3^n}{2^n}$

(b) $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

(c) $\sum_{n=1}^{\infty}(cos(1))^n$

17. **(10 points)** Find a power series expansion for the function $f(x) = \dfrac{1}{x}$ with center $a = -3$. What is its radius of convergence?

18. **(10 points total, 5 points each)**

(a) Show that the series $\sum_{n=1}^{\infty}(-1)^{n-1}ne^{-n}$ is convergent.

(b) How many terms do we need to add in order to find the sum correct to one decimal places (that is, so that $|$ error $|< 0.1$)? (You don't need to calculate the estimated sum).

19. **(10 points)** Find the equation of the plane that contains the point $(1, 3, 1)$ and the line $x = t$, $y = t$, and $z = t + 2$.