# Homework #1 for MATH 8302: Manifolds and Topology II

February 5, 2018

**Due Date:** Monday 12 February in class.

1. (a) Let $p \in \mathbb{R}P^n$ be any point, and let $D \subseteq \mathbb{R}P^n$ be a small open disk containing $p$; show that $\mathbb{R}P^n \setminus D$ is homotopy equivalent to $\mathbb{R}P^{n-1}$.

   (b) Let $S = \partial(\overline{D})$ be the boundary of the closure of $D$; $S$ is homeomorphic to $S^{n-1}$. Compute the map $H_*(S) \to H_*(\mathbb{R}P^n \setminus D)$ which is induced by the inclusion of the subspace.

   (c) Compute $H_*(\mathbb{R}P^n \# \mathbb{R}P^n)$.

2. (a) Find a representative for a generator of $\widetilde{H}_0(S^0) \cong \mathbb{Z}$.

   (b) Enumerate the set of continuous maps $S^0 \to S^0$. Using the previous problem, compute their degrees.

3. Show that every self map of $\mathbb{R}P^n$ has a fixed point if $n$ is even. Is the same true if $n$ is odd?

4. Let $X$ be the quotient space of the cube $I^3$ where one identifies opposite faces with a 1/4 twist.

   (a) Show that $X$ supports a CW structure with two 0-cells, four 1-cells, three 2-cells, and one 3-cell.

   (b) Compute $H_*(X)$ using cellular homology.