Tutorial Info:

- **Website:** http://ms.mcmaster.ca/~dedieula.
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
- **Email:** dedieula@math.mcmaster.ca.
Examples:

1. Consider the following sets of vectors:

\[ S_1 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix} \right\}, S_2 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \right\}, \]

\[ S_3 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \right\}. \]
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a) Which sets span \( \mathbb{R}^3 \)?
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a) Which sets span \( \mathbb{R}^3 \)?

Recall: The span of a set \( S = \{w_1, \ldots, w_r\} \), is the subspace formed by taking all possible linear combinations of the vectors in \( S \). i.e.

\[ \text{span}(S) = \{ \alpha_1 w_1 + \ldots + \alpha_r w_r \mid \alpha_1, \ldots, \alpha_r \in \mathbb{R} \}. \]
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Recall: If \( A \) is square, then \( Ax = b \) is consistent for every \( n \times 1 \) matrix \( b \iff \det(A) \neq 0 \).
Examples:

b) Is the vector
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\begin{pmatrix}
3 \\
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\end{pmatrix}
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in the span of \( S_1 \)? \( S_2 \)? \( S_3 \)?
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b) Is the vector
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c) Which of these sets are linearly independent?
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Recall: If a set of vectors \( S = \{v_1, \ldots, v_r\} \) is such that the equation
\[
\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_r v_r = \vec{0}
\]
has only the trivial solution (i.e. \( \alpha_1 = \ldots = \alpha_r = 0 \)), then these vectors are said to be **linearly independent**. If there exist nontrivial solutions, then the vectors are said to be **linearly dependent**.
Examples:

- 2. Which of the following form a basis for $\mathbb{R}^2$?
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2. Which of the following form a basis for $\mathbb{R}^2$?

Recall: A set $S = \{v_1, \ldots, v_n\}$ of vectors, where $v_1, \ldots, v_n \in V$ is called a basis for $V$ if:

- The vectors in $S$ are linearly independent.
- $S$ spans $V$.

S = \{(1, 0), (0, 1)\} is called the standard basis for $\mathbb{R}^2$.

If \{\(v_1, \ldots, v_n\)\} is a basis for $V$ then:

- If a set $S$ of vectors from $V$ has $n+1$ vectors, then $S$ is linearly dependent.
- If $S$ has $n-1$ vectors, then $S$ does not span $V$.

a) $S = \{(1, 0), (1, 1), (0, 2)\}$.

b) $T = \{(1, 1), (1, -1)\}$. 
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1. If a set $S$ of vectors from $V$ has $> n$ vectors, then $S$ is linearly dependent.
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- b) Find the vector \( w \in \mathbb{R}^2 \) whose coordinate vector relative to \( T \) is \( [w]_T = (4, 2) \).
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4.) Which of the following are a basis for $P_2$ (where $P_2$ is the vector space of all polynomials of degree $\leq 2$; i.e. $P_2 = \{a + bx + cx^2| a, b, c \in \mathbb{R}\}$.
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