Tutorial Info:

- **Website:** http://ms.mcmaster.ca/~dedieula.
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
- **Email:** dedieula@math.mcmaster.ca.
Does the Commutative Law for Multiplication hold for Matrices?, i.e. is it always true that $AB = BA$?
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- e.g. If

  \[ A = \begin{pmatrix} 3 & 1 \\ 2 & 1 \\ 2 & 3 \end{pmatrix}, B = \begin{pmatrix} 3 & 0 & -1 & -1 \\ 1 & 2 & 3 & -1 \end{pmatrix} \]

  then

  \[ AB = \begin{pmatrix} 10 & 2 & 0 & -4 \\ 7 & 2 & 1 & -3 \\ 9 & 6 & 7 & -5 \end{pmatrix}, \]

  but $BA$ is not defined.
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$$AB = \begin{pmatrix} 10 & 2 & 0 & -4 \\ 7 & 2 & 1 & -3 \\ 9 & 6 & 7 & -5 \end{pmatrix},$$

but $BA$ is not defined.

- So no, it is not true in general that $AB = BA$. 


Does the Commutative Law for Multiplication hold for Matrices?

- What if $A$ and $B$ are both square (i.e. $A$ and $B$ are both $n \times n$ matrices)?
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- What if $A$ and $B$ are both square (i.e. $A$ and $B$ are both $n \times n$ matrices)?
- Does $AB = BA$ for any possible $A$ and $B$?
- Can you think of a counterexample?
Does the Commutative Law for Multiplication hold for Matrices?

- Is it ever possible to find an $A$ and $B$ such that $AB = BA$?
Zero Divisors?

- For real numbers, we know that \( ab = 0 \Rightarrow a = 0 \) or \( b = 0 \).
Zero Divisors?

- For real numbers, we know that $ab = 0 \Rightarrow a = 0$ or $b = 0$.

- Is this true for matrices? (i.e. if we have two matrices $A$ and $B$ such that $AB = 0$, is it true that we must have $A = 0$ or $B = 0$?)
Cancellation Law?

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Cancellation Law?

- For real numbers, we know that $ab = ac \Rightarrow b = c$.
- Does this hold true in general for matrices? (i.e. $AB = AC \Rightarrow B = C$?)
Recap: In general, it is not true that:

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Recap: In general, it is not true that:

- $AB = BA$ (i.e. multiplicative commutativity fails)
- $AB = 0 \Rightarrow A = 0$ or $B = 0$ (i.e. $\exists$ non-zero zero divisors)
- $AB = AC \Rightarrow B = C$ (i.e. cancellation law fails)
Multiplicative Identity

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- For matrices, this "1" is known as the identity matrix, e.g. if $A$ is $m \times n$, then $A \times I_{n \times n} = A$.
- e.g.
Multiplicative Inverse

- In $\mathbb{R}$ we know that for every $a$ such that $a \neq 0$ there exists $a^{-1}$ such that $aa^{-1} = a^{-1}a = 1$. 

- If $A$ is a square ($n \times n$) matrix such that $\exists B$ such that $AB = I_{n \times n} = BA$, then $A$ is said to be invertible (a.k.a nonsingular), and $B$ is called the inverse of $A$, ($B = A^{-1}$).

- If $A$ is a $2 \times 2$ matrix, then $A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.
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- If $A$ is a $2 \times 2$ matrix, then

$$A^{-1} = \frac{1}{ad - bc} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix},$$

b/c:
For 2x2 Matrices:

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- $\text{det}(A) = ad - bc$.
- $A$ is nonsingular $\iff \text{det}(A) \neq 0$.
- So, $\text{det}(A) = 0 \iff A$ is singular (i.e. $A$ is not invertible).
Examples:

1. Let

\[ A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix}. \]
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b) Find \( A^{-1} \).
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- a) Is \( A \) invertible?.
- b) Find \( A^{-1} \).
- c) Is \( B \) invertible?.
Examples:

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a) Is \( A \) invertible?.

b) Find \( A^{-1} \).

c) Is \( B \) invertible?.

d) Find \( B^{-1} \).
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1. Let

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- a) Is \( A \) invertible?.
- b) Find \( A^{-1} \).
- c) Is \( B \) invertible?.
- d) Find \( B^{-1} \).
- e) Find \( (AB)^{-1} \).
Examples:

2. Let

\[ A = \begin{pmatrix} 4 & x \\ x & 1 \end{pmatrix}. \]

For what values of \( x \) is \( A \) singular?
Examples:

3. Solve for $X$: $A(X + B) = CA$ (where $A$ is invertible).
Examples:

- 4. Solve for $X$: $(2E + F)^T = G^{-1}X^T + F^T$. 
Examples:

5. Find the inverse of

\[ A = \begin{pmatrix} 1 & 1 & 1 \\ 6 & 7 & 5 \\ 3 & 2 & 3 \end{pmatrix} \]

using row operations.
Examples:

6. a) Solve for $W$: $2EWF^2 = (E^TF)^2$. 
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b) What sizes must $F$ and $W$ be in order for $W$ to have a unique solution if $E$ is $3 \times n$?