Tutorial Info:

- **Website:** http://ms.mcmaster.ca/~dedieula.
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
- **Email:** dedieula@math.mcmaster.ca.
Examples:

1. Consider

\[ A = \begin{pmatrix} 8 & 9 \\ -6 & -7 \end{pmatrix}. \]

a) What are the eigenvalues of \( A \)?.
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So, since we’re looking for vectors \( x \) such that \( (A - \lambda I)x = 0 \) and we know that \( x \neq 0 \) by definition, then by our equivalent statements about inverses that must mean that \( \det(A - \lambda I) = 0. \)
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So, \( \lambda \) is an eigenvalue of \( A \) \( \iff \) it satisfies the equation \( \det(A - \lambda I) = 0 \).
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- b) Find all eigenvectors of $A$. 
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- 2.a) Find all eigenvalues of

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A = \begin{pmatrix}
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-1 & -4 & 5 \\
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■ b) Find all eigenvectors corresponding to \( \lambda = -3 \).
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b) Find all eigenvectors corresponding to \( \lambda = -3 \).

c) Is \( A \) invertible?
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- 3.a) Find the eigenvalues of

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- b) Find all eigenvectors corresponding to \( \lambda = 2 \).
4.) Consider

\[ A = \begin{pmatrix} 5 & -3 \\ a & b \end{pmatrix} \]

and suppose

\[ x = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \]

is an eigenvector of \( A \). What must the eigenvalue \( \lambda \) corresponding to \( x \) be?
Examples:

5.) Find all eigenvalues and eigenvectors of $A^{10}$, if

$$A = \begin{pmatrix} 8 & 9 \\ -6 & -7 \end{pmatrix}.$$