Tutorial Info:

- **Website:** http://ms.mcmaster.ca/~dedieula.
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
- **Email:** dedieula@math.mcmaster.ca.
Examples:

1. Suppose the population of raccoons in the city in 2010 is 100 and the population of raccoons in the nearby forest is 300. Suppose we also know that 10% of the raccoons in the forest move to the city, and 5% of the raccoons in the city move to the forest each year.
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   Recall: Our transition matrix takes us from time \( k \) to time \( k+1 \):

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   b) Is \( T \) a regular stochastic matrix?

   Recall: A square matrix \( A \) is called a stochastic matrix if each of its columns is a probability vector (i.e. the entries of each column sum to 1).

   c) Does \( T \) have a steady-state vector? If so, what is it?

   Recall: If \( P \) is a regular transition matrix for a Markov chain, then \( \exists ! \) probability vector \( q \) such that \( Pq = q \) (i.e. \( q \) is an eigenvector corresponding to \( \lambda = 1 \) and \( q \)’s entries sum to 1). This vector is called the steady-state vector.
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Recall: A stochastic matrix $A$ is called **regular** if $A$, or some positive power of $A$, has all positive entries.

**d)** In the long term, how will the population of raccoons in the city and woods be distributed?

Recall: If $q$ is a steady-state vector for a regular Markov chain, then for any initial probability vector $x_0$, $\lim_{k \to \infty} P^k x_0 = q$, where $P$ is the transition matrix for this chain.

**e)** How many raccoons will be in the city after 20 years?

Recall: We know $x_n = P^n x_0$, where $x_0$ is the initial state vector, $x_n$ is the state vector at time $n$, and $P$ is the transition matrix.
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Examples:

2. Express $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$ as a real number.
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- 3. Consider \( z = \frac{i}{-2-2i} \).
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a) Express \( z \) in rectangular form.
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b) Express $z$ in polar form.

c) What is $\text{Arg} \ z$?

Recall: The argument of $z$ is multivalued, i.e. $\text{arg} \ z = \theta + 2\pi k, k \in \mathbb{Z}$.

The principal argument, $\text{Arg} \ z$, is such that $-\pi < \text{Arg} \ z \leq \pi$. 
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■ The **principal argument**, \( \text{Arg} z \), is such that \(-\pi < \text{Arg} z \leq \pi \).

■ d) What is \( \bar{z} \)?

■ **Recall:** If \( z = a + bi \), then the **complex conjugate** of \( z \) is: \( \bar{z} = a - bi \).
Examples:

4. Express $(\sqrt{3} - i)^6$ in polar form.
Examples:

- 5. Find the solutions to the equation $z^3 = -1$. 
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- **Recall:** 
  
  $z^{\frac{1}{n}} = \sqrt[n]{r}[\cos\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)], \ k = 0, 1, \ldots, n - 1$. 
Examples:

- 6. a) Find the square roots of $2i$. 
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- b) Express your two roots in rectangular coordinates.