Tutorial Info:

- **Website:** http://ms.mcmaster.ca/~dedieula.
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
- **Email:** dedieula@math.mcmaster.ca.
Examples:

- 1. Find a unit vector that has the same direction as $(-4, -3)$. 
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Recall: a vector of norm 1 is called a unit vector. i.e. if \(\|u\| = 1\), then \(u\) is a unit vector.
Examples:

2. Let \( u = (0, 2, 2, 1) \) and \( v = (1, 1, 1, 1) \). Verify that the Cauchy-Schwartz inequality holds.
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Recall: Cauchy-Schwartz Inequality: \( |u \cdot v| \leq ||u|| ||v|| \).
Examples:

3. Suppose $||u|| = 2, ||v|| = 1$, and $u \cdot v = 1$. What is the angle in radians between $u$ and $v$?
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- **3.** Suppose $||u|| = 2, ||v|| = 1$, and $u \cdot v = 1$. What is the angle in radians between $u$ and $v$?

- **Recall:** $\cos \theta = \frac{u \cdot v}{||u|| ||v||}$. 
Examples:

4. Let \( u = (1, 0, 1) \) and \( v = (0, 1, 1) \).
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- 4. Let $u = (1, 0, 1)$ and $v = (0, 1, 1)$.
- a) Find two unit vectors orthogonal to both $u$ and $v$. 

Recall:

- Two vectors $u$ and $v$ are orthogonal if $u \cdot v = 0$.
- A nonempty set of vectors in $\mathbb{R}^n$ is called an orthogonal set if all pairs of distinct vectors in the set are orthogonal.
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Recall: Two vectors $u$ and $v$ are **orthogonal** if $u \cdot v = 0$.

b) Do $u$, $v$, and one of the unit vector you found form an orthogonal set?
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Examples:

- 5. What does the equation \(-2(x + 1) + (y - 3) - (z + 2) = 0\) represent geometrically?
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- Recall: The **point normal equation** of a plane is:
  \[ a(x - x_0) + b(y - y_0) + c(z - z_0) = 0, \]
  where \(P_0(x_0, y_0, z_0)\) is a specific point on the plane, \(P = (x, y, z)\) is an arbitrary point on the plane, and \(n = (a, b, c)\) is the normal vector to the plane.
Examples:

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Recall: If $u$ and $a$ are vectors in $\mathbb{R}^n$ such that $a \neq 0$, then we can write $u = w_1 + w_2$, where $w_1 = \text{proj}_a u = \frac{u \cdot a}{||a||^2} a$ (vector component of $u$ along $a$; a.k.a. orthogonal projection of $u$ along $a$), and $w_2 = u - w_1 = u - \text{proj}_a u$ (component of $u$ orthogonal to $a$).
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b) Find the vector component of $u$ orthogonal to $a$. 
Examples:

7. Find the distance between the point \((3, 1, -2)\) and the plane \(x + 2y - 2z = 4\).
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- 7. Find the distance between the point \((3, 1, -2)\) and the plane \(x + 2y - 2z = 4\).
- **Recall:** In \(\mathbb{R}^3\), the distance between a point \(P_0(x_0, y_0, z_0)\) and a plane \(ax + by + cz + d = 0\) is: \[
\frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}.
\]
8. Consider two points $P(2, 3, -2)$ and $Q(7, -4, 1)$. Find the point on the line segment containing $P$ and $Q$ that is $\frac{3}{4}$ of the way from $P$ to $Q$. 
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Recall: The vector with initial point \( P_1(x_1, y_1, z_1) \) and terminal point \( P_2(x_2, y_2, z_2) \) is given by the formula: \( \overrightarrow{P_1P_2} = (x_2 - x_1, y_2 - y_1, z_2 - z_1) \).
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Recall: The vector equation of a line $\ell$ through the point $x_0$ that is parallel to $v$ is $\ell = x_0 + tv$. i.e. $v$ gives direction and $x_0$ gives position.
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- c) Which line passes through $(1, 2)$ and is parallel to $\ell$?

- Recall: Two lines are parallel if their direction vectors are multiples of each other.

- d) Find a line that is perpendicular to $\ell$.

- Recall: Two lines are perpendicular if their dot product is zero.
Examples:

10. Find a vector equation of the plane in $\mathbb{R}^4$ passing through the point $(2, -1, 7, 3)$ and parallel to both $(1, 0, 2, 1)$ and $(3, 2, 4, 5)$.
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- **10.** Find a vector equation of the plane in $\mathbb{R}^4$ passing through the point $(2, -1, 7, 3)$ and parallel to both $(1, 0, 2, 1)$ and $(3, 2, 4, 5)$.

- **Recall:** The equation of a plane passing through a point $x_0$ and parallel to $\mathbf{v}_1$ and $\mathbf{v}_2$, where $\mathbf{v}_1$ and $\mathbf{v}_2$ are not collinear, is $x = x_0 + \mathbf{v}_1 t + \mathbf{v}_2 s$. 
Examples:

- 11. Find the area of the triangle with vertices $P = (1, 1, 5)$, $Q = (3, 4, 3)$, and $R = (1, 5, 7)$. 
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- **Recall:**

\[
\begin{align*}
\mathbf{u} \times \mathbf{v} &= \begin{vmatrix} 
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
u_1 & u_2 & u_3 \\
v_1 & v_2 & v_3 
\end{vmatrix},
\end{align*}
\]

where \( \mathbf{i}, \mathbf{j}, \) and \( \mathbf{k} \) are the standard unit vectors

\[
\mathbf{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.
\]