Tutorial Info:

- **Website:** [http://ms.mcmaster.ca/~dedieula](http://ms.mcmaster.ca/~dedieula).
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
- **Email:** dedieula@math.mcmaster.ca.
Examples:

1. Consider the following sets of vectors:

\[ S_1 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix} \right\}, \quad S_2 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \right\}, \quad S_3 := \left\{ \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \right\}. \]

a) Which sets span \( \mathbb{R}^3 \)?

Recall: The span of a set \( S = \{w_1, \ldots, w_r\} \), is the subspace formed by taking all possible linear combinations of the vectors in \( S \). i.e.
\[ \text{span}(S) = \{ \alpha_1 w_1 + \ldots + \alpha_r w_r \mid \alpha_1, \ldots, \alpha_r \in \mathbb{R} \}. \]

Recall: If \( A \) is square, then \( Ax = b \) is consistent for every \( n \times 1 \) matrix \( b \) \(\iff\) \( \det(A) \neq 0 \).
Examples:

- **b)** Is the vector
  \[
  \begin{pmatrix}
  3 \\
  -1 \\
  2
  \end{pmatrix}
  \]
  in the span of \( S_1 \)? \( S_2 \)? \( S_3 \)?

- **c)** Which of these sets are linearly independent?

**Recall:** If a set of vectors \( S = \{v_1, \ldots, v_r\} \) is such that the equation
\[
\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_r v_r = \overline{0}
\]
has only the trivial solution (i.e. \( \alpha_1 = \ldots = \alpha_r = 0 \)), then these vectors are said to be **linearly independent**. If there exist nontrivial solutions, then the vectors are said to be **linearly dependent**.
Examples:

- 2. Which of the following form a basis for \( \mathbb{R}^2 \)?

- **Recall:** A set \( S = \{v_1, \ldots, v_n\} \) of vectors, where \( v_1, \ldots, v_n \in V \) is called a **basis** for \( V \) if:
  1. The vectors in \( S \) are linearly independent.
  2. \( S \) spans \( V \).

- \( S = \{(1, 0), (0, 1)\} \) is called the **standard basis** for \( \mathbb{R}^2 \).

- If \( \{v_1, \ldots, v_n\} \) is a basis for \( V \) then:
  1. If a set \( S \) of vectors from \( V \) has \( > n \) vectors, then \( S \) is linearly dependent.
  2. If \( S \) has \( < n \) vectors, then \( S \) does not span \( V \).

- **a)**
  \[
  S := \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} \right\}.
  \]

- **b)**
  \[
  T := \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}.
  \]
Examples:

3. We know

\[ T := \left\{ \begin{pmatrix} 1 \\ 1 \\ \end{pmatrix} , \begin{pmatrix} 1 \\ -1 \\ \end{pmatrix} \right\} \]

forms a basis for \( \mathbb{R}^2 \).

a) Find the coordinate vector of \( v = (3, 5) \) relative to the basis \( T \). i.e. Find \([v]_T\).

Recall: If \( S = \{ v_1, \ldots, v_n \} \) is a basis for \( V \), and \( w = k_1 v_1 + k_2 v_2 + \ldots + k_n v_n \) for \( k_1, \ldots, k_n \in \mathbb{R} \), then \([w]_S = (k_1, \ldots, k_n)\) is called the coordinate vector of \( v \) relative to \( S \).

b) Find the vector \( w \in \mathbb{R}^2 \) whose coordinate vector relative to \( T \) is \([w]_T = (4, 2)\).
Examples:

4.) Which of the following are a basis for $P_2$ (where $P_2$ is the vector space of all polynomials of degree $\leq 2$; i.e. $P_2 = \{ a + bx + cx^2 | a, b, c \in \mathbb{R} \}$.

- $W = \{ x^2, x + 1, x^2 + x + 1 \}$
- $X = \{ x, 1, 0 \}$
- $T = \{ x^2 + x + 1, x^2 + x, x + 1 \}$
- $Y = \{ x^2, x, 1, 0 \}$