Tutorial Info:

- **Website:** http://ms.mcmaster.ca/~dedieula.
- **Math Help Centre:** Wednesdays 2:30-5:30pm.
- **Email:** dedieula@math.mcmaster.ca.
Examples:

1. Find a unit vector that has the same direction as \((-4, -3)\).

Recall: a vector of norm 1 is called a unit vector. i.e. if \(|u| = 1\), then \(u\) is a unit vector.
Examples:

- 2. Let \( u = (0, 2, 2, 1) \) and \( v = (1, 1, 1, 1) \). Verify that the Cauchy-Schwartz inequality holds.

- Recall: Cauchy-Schwartz Inequality: \(|u \cdot v| \leq ||u|| ||v||\).
Examples:

3. Suppose $||u|| = 2, ||v|| = 1$, and $u \cdot v = 1$. What is the angle in radians between $u$ and $v$?

Recall: $\cos \theta = \frac{u \cdot v}{||u|| ||v||}$. 
Examples:

4. Let \( u = (1, 0, 1) \) and \( v = (0, 1, 1) \).

a) Find two unit vectors orthogonal to both \( u \) and \( v \).

Recall: Two vectors \( u \) and \( v \) are orthogonal if \( u \cdot v = 0 \).

b) Do \( u \), \( v \), and one of the unit vector you found form an orthogonal set?

Recall: A nonempty set of vectors in \( \mathbb{R}^n \) is called an orthogonal set if all pairs of distinct vectors in the set are orthogonal.
Examples:

- 5. What does the equation $-2(x + 1) + (y - 3) - (z + 2) = 0$ represent geometrically?

- **Recall:** The **point normal equation** of a plane is:
  
  $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$, where $P_0(x_0, y_0, z_0)$ is a specific point on the plane, $P = (x, y, z)$ is an arbitrary point on the plane, and $n = (a, b, c)$ is the normal vector to the plane.
Examples:

6. Let \( u = (6, 2) \) and \( a = (3, -9) \).

a) Find the vector component of \( u \) along \( a \).

Recall: If \( u \) and \( a \) are vectors in \( \mathbb{R}^n \) such that \( a \neq 0 \), then we can write \( u = w_1 + w_2 \), where \( w_1 = \text{proj}_a u = \frac{u \cdot a}{||a||^2} a \) (vector component of \( u \) along \( a \); a.k.a. orthogonal projection of \( u \) along \( a \)), and \( w_2 = u - w_1 = u - \text{proj}_a u \) (component of \( u \) orthogonal to \( a \)).

b) Find the vector component of \( u \) orthogonal to \( a \).
Examples:

7. Find the distance between the point \((3, 1, -2)\) and the plane \(x + 2y - 2z = 4\).

Recall:) In \(\mathbb{R}^3\), the distance between a point \(P_0(x_0, y_0, z_0)\) and a plane \(ax + by + cz + d = 0\) is: \[
\frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}.
\]
Examples:

- 8. Consider two points $P(2, 3, -2)$ and $Q(7, -4, 1)$. Find the point on the line segment containing $P$ and $Q$ that is $\frac{3}{4}$ of the way from $P$ to $Q$.

- Recall: The vector with initial point $P_1(x_1, y_1, z_1)$ and terminal point $P_2(x_2, y_2, z_2)$ is given by the formula: $\overrightarrow{P_1P_2} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$.
Examples:

9. a) What is the vector equation of the line $4y + 3x = 40$?

Recall: The vector equation of a line $\ell$ through the point $x_0$ that is parallel to $v$ is $\ell = x_0 + tv$. i.e. $v$ gives direction and $x_0$ gives position.

b) What are the parametric equations of this line?

c) Which line passes through $(1, 2)$ and is parallel to $\ell$?

Recall: Two lines are parallel if their direction vectors are multiples of each other.

d) Find a line that is perpendicular to $\ell$.

Recall: Two lines are perpendicular if their dot product is zero.
Examples:

- **10.** Find a vector equation of the plane in $\mathbb{R}^4$ passing through the point $(2, -1, 7, 3)$ and parallel to both $(1, 0, 2, 1)$ and $(3, 2, 4, 5)$.

- **Recall:** The equation of a plane passing through a point $x_0$ and parallel to $v_1$ and $v_2$, where $v_1$ and $v_2$ are not collinear, is $x = x_0 + v_1 t + v_2 s$. 
Examples:

- 11. Find the area of the triangle with vertices \( P = (1, 1, 5) \), \( Q = (3, 4, 3) \), and \( R = (1, 5, 7) \).

- **Recall:** If \( u \) and \( v \) are vectors in 3-space, then \( ||u \times v|| = \text{area of the parallelogram determined by } u \text{ and } v \).

- **Recall:**

\[
\begin{align*}
\mathbf{u} \times \mathbf{v} &= \begin{vmatrix}
i & j & k \\
u_1 & u_2 & u_3 \\
v_1 & v_2 & v_3 \\
\end{vmatrix},
\end{align*}
\]

where \( i, j, \) and \( k \) are the standard unit vectors:

\[
\begin{align*}
i &= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \\
j &= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \\
k &= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.
\end{align*}
\]