2.1: Solution Curves Without a Solution

#1, 15, 21, 23, 25, 29

1. Sketch by hand an approx. solution curve that passes through each of the indicated points.

- $y(0) = 1$
- $y(1) = 0$
- $y(2) = 0$
- $y(0) = 0$

*See textbook for what the given direction field looks like.*

15. Sketch isolines $F(x, y) = c$ for the given DE. Use this to sketch the direction field of a solution curve for $y' = y^2$.

Recall: Given a DE $y' = F(x, y)$, an isoline is any member of the family of curves $F(x, y) = c, c \in \mathbb{R}$.

@ $\frac{dy}{dx} = x + y; \ c \in \mathbb{R}, -5 \leq c \leq 5$.

Here $F(x, y) = x + y$. For each $c$, $x + y = c \Rightarrow y = -x + c$ is a straight line w/ slope $-1$.
\( \frac{dy}{dx} = x^2 + y^2 \) 

\( c = \sqrt{c} \) 

\( c = 4 \) 

\( c = 9 \) 

\( c = 1 \) 

\( x^2 + y^2 = c \) is a circle of radius \( \sqrt{c} \).

The slope of the direction field along each isocline corresponds to \( \frac{dy}{dx} = \sqrt{c} \) at each critical point.

\# 21, 33, 35: Find critical pts & phase portrait. Classify each critical pt as asymptotically stable, unstable, or semi-stable. Sketch typical solution curves.

21. \( \frac{dy}{dx} = y^2 - 3y \)

\( f(y) = y^2 - 3y \)

\( f(0) = 0 \) 

\( f(3) = 6 \) 

\( f(-2) = -10 \) 

\( f(4) = 10 \)

\( 5, y \) is negative.

The phase portrait is:

\( 0 \) is stable & \( 3 \) is unstable.

\( f(y) \)

\( 4 \)

\( -2 \)

\( 4 \)

\( y \) vs \( x \)
23. \( \frac{dy}{dx} = (y-a)^4 \). One critical pt \( y=2 \).

\[
\begin{array}{c|ccc}
F(y) & 0 & 3 & 1 \\
\text{sign} & + & + & +
\end{array}
\]

25. \( \frac{dy}{dx} = y^2(4-y^2) \). \( y^2(4-y^2) = 0 \Leftrightarrow y=0 \text{ or } y=2 \text{ or } y=-2 \).

\[
\begin{array}{cccccc}
-3 & 0 & 1 & 0 & 1 & 2 & 3 \\
- & + & + & + & - & + & -
\end{array}
\]

29. Consider the autonomous DE \( \frac{dy}{dx} = F(y) \), where the graph of \( F \) is given. Locate the critical pts of the DE, sketch a phase portrait & sketch typical solution curves.

Critical pts are 0 & c.

\[
\begin{array}{ccc}
-1 & 0 & c+1 \\
+ & - & +
\end{array}
\]

Looking to see if \( F \) is pos. or neg.

\[
\begin{array}{c}
0 \text{ stable} \\
& c \text{ unstable.}
\end{array}
\]