Tutorial Info:

- **Tutorial Website:** http://ms.mcmaster.ca/~dedieula/2Z03.html
- **Office Hours:** Mondays 3pm - 5pm (in the Math Help Centre)
Tutorial #4:

- 3.1 Theory of Linear Equations
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  - Linearly Independence
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- 3.3 Homogeneous Linear Equations with Constant Coefficients
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  - Finding Rational Roots
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- 3.3 Homogeneous Linear Equations with Constant Coefficients
  - Finding Rational Roots
  - Finding Complex Roots
3.1 Theory of Linear Equations

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- **Recall:** A set of functions \( f_1(x), \ldots, f_n(x) \) are **linearly independent** on an interval \( I \) if \( c_1f_1(x) + \cdots + c_nf_n(x) = 0 \) for all \( x \) in \( I \) \iff \( c_1 = \cdots c_n = 0 \).
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Criterion for Linear Independent Solutions: Let \( y_1, \ldots, y_n \) be solutions of a homogeneous linear \( n \)-th order DE on an interval \( I \). Then this set of solutions is linearly independent on \( I \) if and only if the Wronskian \( W(y_1, \ldots, y_n) \neq 0 \) for every \( x \) in \( I \).
2. Suppose $f_1$, $f_2$, and $f_3$ are solutions to a second-order linear homogeneous differential equation. Is \( \{f_1, f_2, f_3\} \) a fundamental set of solutions?
3.1 Theory of Linear Equations

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- **Recall:** A basis for the space of solutions of an $n$-th order homogenous linear equation $a_n y^{(n)} + \cdots + a_1 y' + a_0 = 0$ is called a fundamental set of solutions. 

  We know the dimension of the solution space is $n$, so to find a basis, it suffices to find $n$ linearly independent solutions.
3.1 Theory of Linear Equations

- 3. The functions $e^t$ and $te^t$ satisfy the differential equation $y'' - 2y' + y = 0$. Is $y = c_1 e^t + c_2 te^t$ a general solution of this differential equation?
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**Recall:** If $\{y_1, \ldots, y_n\}$ is a fundamental set of solutions on $I$ for an $n$-th order linear DE, then the **general solution** on $I$ is $y = c_1y_1 + \cdots c_ny_n$, where the $c_i$ are arbitrary constants.
3.3 Homogenous Linear Equations with Constant Coefficients:

- 4. Find the general solution of $y^{(3)} + 8y = 0$. 

Recall: To solve, first we plug $y = e^{mx}$ into the equation and find the roots of the corresponding auxiliary equation. We can go about finding the roots of this auxiliary equation in a variety of ways.
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3.3 Homogenous Linear Equations with Constant Coefficients:

- **Method 1**: Find one root \( d \), then divide \( m^3 + 8 \) by \( (m - d) \).
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3.3 Homogenous Linear Equations with Constant Coefficients:

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- **Rational Roots Test (pg. 122):** If $m_1 = \frac{p}{q}$ is a rational root (expressed in lowest terms) of an auxiliary equation with integer coefficients

  \[ a_nm^n + \ldots + a_1m + a_0, \]

  then $p$ is a factor of $a_0$ and $q$ is a factor of $a_n$. 
3.3 Homogeneous Linear Equations with Constant Coefficients:

- **Method 2: Finding roots over \( \mathbb{C} \):** Given auxiliary equation

  \[
  a_n m^n + \ldots + a_1 m + a_0,
  \]

  we know there will be \( n \) roots over the complex numbers (with some multiplicity). By working in polar coordinates, we can find these roots \( re^{i\theta} \) (think back to Math 1ZC3/1B03).
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- **Exercise** Find the five roots of \( m^5 + 32 = 0 \), and use this to solve \( y^{(5)} + 32y = 0 \).
3.3 Homogenous Linear Equations with Constant Coefficients:

- Recall:
  - If there are $j$ distinct roots $m_1, \ldots, m_j$ then the general solution contains a linear combination of $e^{m_1x}, \ldots, e^{m_jx}$.
  - If $m_1$ is a root of multiplicity $q$, then the general solution contains a linear combination of $e^{m_1x}, xe^{m_1x}, x^2e^{m_1x}, \ldots, x^{q-1}e^{m_1x}$.
  - Given complex roots $m_1 = \alpha + \beta$ and $m_2 = \alpha - \beta$, using Euler’s formula, it’s always possible to write $c_1e^{m_1x} + c_2e^{m_2x}$ as $e^{\alpha x} [k_1\cos(\beta x) + k_2\sin(\beta x)]$, for constants $c_1, c_2, k_1, k_2$. 
3.3 Homogenous Linear Equations with Constant Coefficients:

- 5. Solve the IVP $y'' + 2y' = 0$, $y(0) = 1$, $y'(0) = 1$. 
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- 6. Find a general solution for $6y^{(4)} - y''' + 4y'' - y' - 2y = 0$. 