Bond returns

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Yield

I do not want to encourage you to associate yield with return. Yield is merely a convenient way of generalizing about bond prices. Return in contrast is about the performance of an asset. Let’s translate a timeseries of bond yields into a timeseries of daily log returns.

Often when we have a yield timeseries, we do not also have historical information about the term of the actual bond or bonds. So we cannot reverse engineer the actual price data. In fact, sometimes the reported yield has been interpolated from several bonds, so in fact there is no precise bond. The Federal Reserve’s H.15 report is an example of this. This data includes of “constant maturity” yields from the U. S. Treasuries market at various tenors, two years, five years, ten years, etc.

Let’s consider a bond paying equal semi-annual interest. Let the maturity date be $T$ and the annual coupon rate be $c$. The relationship between price and yield is given by

$$P(t) = \frac{c}{y(t)} \left(1 + \frac{y(t)}{2}\right)^{2(t-t_0)} + \left(1 - \frac{c}{y(t)}\right) \left(1 + \frac{y(t)}{2}\right)^{-2(T-t+t_0)}$$

where $t_0$ is six months before the next coupon after $t$. This representation assumes coupon payments are equally spaced. The Treasury’s yield calculation is slightly different, but these differences are immaterial—especially considering that the H.15 data is rounded to the nearest basis point.

Note that if we assume $c = y(t_0)$, then $P(t_0) = 1$. That is, the bond was issued at par. This is a good approximation for most bonds, whose coupons are set with this goal in mind.

Return

Let’s interpret the yield in the next period firstly as the new yield on the previous period’s par bond, and secondly as the coupon on the next period’s par bond. The value of the former bond is

$$P(t + \Delta t) = \left(1 + \frac{y + \Delta y}{2}\right)^{2\Delta t} \frac{1 + \frac{\Delta y}{y}}{1 + \frac{\Delta y}{y}} \left(1 + \frac{y + \Delta y}{2}\right)^{-2T}$$

(1)

Since the initial value was one, the log-return on this bond is just the log of $P(t + \Delta t)$. While this is simple enough to calculate “exactly”, a first-order approximation is very accurate

$$\log \left[ \frac{P(t + \Delta t)}{P(t)} \right] \approx y \Delta t - \frac{1 - (1 + \frac{y}{2})^{-2T}}{y} \Delta y$$

(2)