Let us try out the two-step approach to determine the optimal portfolio under a single affine constraint with objective linear in the market vector and index of satisfaction equal to the Cornish-Fisher expansion of the 95% expected shortfall.

For an objective defined by $\Psi_\alpha = \alpha' M$ and an affine constraint defined by $d' \alpha = c$, the analytic solution to the optimal mean-variance portfolio satisfies

$$\alpha(\beta) = (1 - \beta) \alpha_{MV} + \beta \alpha_{SR} \quad (1)$$

for $\beta > 0$, where

$$\alpha_{MV} = \frac{c (\text{Cov} M)^{-1} d}{d' (\text{Cov} M)^{-1} d}$$

$$\alpha_{SR} = \frac{c (\text{Cov} M)^{-1} EM}{d' (\text{Cov} M)^{-1} EM}$$

We need to determine the level of $\beta$ that maximizes the index of satisfaction, which we will take to be

$$S(\alpha) = E \Psi_\alpha + \sqrt{\text{Var} \Psi_\alpha} \left( I [\phi \Phi^{-1}] + \frac{1}{6} \left( I \phi (\Phi^{-1})^2 \right) - 1 \right) \text{Skew} \Psi_\alpha$$

based on the Cornish-Fisher expansion, where $\Phi$ is the CDF of a standard normal random variable and $\phi$ is the spectrum for $ES_{0.95}$.

Since we can assume that the skewness of $M$ is negligible, the skewness of $\Psi_\alpha$ is also negligible. Furthermore, we can evaluate the integral in the expansion numerically.

$$I [\phi \Phi^{-1}] = \int_0^{0.05} \frac{\sqrt{2 \text{erf}^{-1}(2p - 1)}}{0.05} dp \approx -2.0627 \cdots$$

Let us assign $z_{0.95} = 2.0627 \cdots$, so the integral above is $-z_{0.95}$. The satisfaction is

$$S(\alpha) = \alpha' EM - z_{0.95} \sqrt{\alpha' (\text{Cov} M) \alpha}$$

Substituting in (1), we get that the optimal value for $\beta$ is

$$\beta^* = \arg \max_{\beta > 0} (1 - \beta) \alpha_{MV} + \beta \alpha_{SR}$$

$$= \frac{(1 - \beta) \alpha_{MV} + \beta \alpha_{SR}}{z_{0.95} \sqrt{(1 - \beta) \alpha_{MV} + \beta \alpha_{SR}}}$$
From manipulation of the first-order condition, recognizing that

$$\text{Cov}(\Psi_{a_{MV}}, \Psi_{a_{SR}} - \Psi_{a_{MV}}) = 0$$

we can determine that the solution is

$$\beta^* = \begin{cases} 0 & \gamma \leq 0 \\ \sqrt{\frac{\text{Var}(\Psi_{a_{MV}})}{\text{Var}(\Psi_{a_{SR}} - \Psi_{a_{MV}})}} (\frac{1}{\gamma})^{1/2} - 1 & 0 < \gamma < z_{0.95} \\ \infty & \gamma \geq z_{0.95} \end{cases}$$

where

$$\gamma = \frac{\text{Var}(\Psi_{a_{SR}} - \Psi_{a_{MV}})}{\text{Var}(\Psi_{a_{SR}} - \Psi_{a_{MV}})}$$

is the market price for risk.

In conclusion, the optimal portfolio is $\alpha(\beta^*)$. 