Let us try out the two-step approach to determine the optimal portfolio under a single affine constraint with objective linear in the market vector and index of satisfaction equal to the Cornish-Fisher expansion of the 95% expected shortfall.

For an objective defined by $\Psi_\alpha = \alpha' M$ and an affine constraint defined by $d' \alpha = c$, the analytic solution to the optimal mean-variance portfolio satisfies

$$\alpha(\beta) = (1 - \beta) \alpha_{MV} + \beta \alpha_{SR} \quad (1)$$

for $\beta > 0$, where

$$\alpha_{MV} = \frac{c (\text{Cov} M)^{-1} d}{d' (\text{Cov} M)^{-1} d}$$

$$\alpha_{SR} = \frac{c (\text{Cov} M)^{-1} \text{EM}}{d' (\text{Cov} M)^{-1} \text{EM}}$$

We need to determine the level of $\beta$ that maximizes the index of satisfaction, which we will take to be

$$S(\alpha) = \text{E} \Psi_\alpha + \sqrt{\text{Var} \Psi_\alpha} \left( \mathcal{I} \left[ \phi \Phi^{-1} \right] + \frac{1}{6} \left( \mathcal{I} \left[ \phi \left( \Phi^{-1} \right)^2 \right] - 1 \right) \text{Skew} \Psi_\alpha \right)$$

based on the Cornish-Fisher expansion, where $\Phi$ is the CDF of a standard normal random variable and $\phi$ is the spectrum for $ES_{0.95}$.

Since we can assume that the skewness of $M$ is negligible, the skewness of $\Psi_\alpha$ is also negligible. Furthermore, we can evaluate the integral in the expansion numerically.

$$\mathcal{I} \left[ \phi \Phi^{-1} \right] = \int_0^{0.05} \frac{\sqrt{2erf^{-1}(2p-1)}}{0.05} dp \approx -2.0627 \cdots$$

Let us assign $z_{0.95} = 2.0627 \cdots$, so the integral above is $-z_{0.95}$. The satisfaction is

$$S(\alpha) = \alpha' \text{EM} - z_{0.95} \sqrt{\alpha' (\text{Cov} M) \alpha}$$

Substituting in (1), we get that the optimal value for $\beta$ is

$$\beta^* = \arg \max_{\beta > 0} \left( (1 - \beta) \alpha_{MV} + \beta \alpha_{SR} \right)' \text{EM}$$

$$- z_{0.95} \sqrt{\left( (1 - \beta) \alpha_{MV} + \beta \alpha_{SR} \right)' (\text{Cov} M) \left( (1 - \beta) \alpha_{MV} + \beta \alpha_{SR} \right)}$$
From manipulation of the first-order condition, recognizing that
\[ \text{Cov} \left( \Psi_{\alpha \text{MV}}, \Psi_{\alpha \text{SR}} - \Psi_{\alpha \text{MV}} \right) = 0 \]
we can determine that the solution is
\[
\beta^* = \begin{cases} 
0 & \gamma \leq 0 \\
\frac{\text{Var} \Psi_{\alpha \text{MV}}}{\text{Var} \left( \Psi_{\alpha \text{SR}} - \Psi_{\alpha \text{MV}} \right) \left( \frac{z_{0.95}}{\gamma} \right)^2 - 1} & 0 < \gamma < z_{0.95} \\
\infty & \gamma \geq z_{0.95}
\end{cases}
\]
where
\[
\gamma = \frac{E \left( \Psi_{\alpha \text{SR}} - \Psi_{\alpha \text{MV}} \right)}{\sqrt{\text{Var} \left( \Psi_{\alpha \text{SR}} - \Psi_{\alpha \text{MV}} \right)}}
\]
is the market price for risk.
In conclusion, the optimal portfolio is \( \alpha(\beta^*) \).