Consider a broad investment universe and a normal market vector. Assume all initial asset prices are one. Under the objective probability measure, each pair-wise correlation is $0 < \rho < 1$ and each component’s mean and variance is $\mu_0$ and $\sigma_0^2$ respectively.

The manager’s subjective probability measure is based on a view with confidence $0 < c < 1$ that one particular market vector component will turn out to be $v_1 \in \mathbb{R}$.

The picks matrix is simply

$$P = \begin{pmatrix} 1 & 0 & 0 & \cdots \end{pmatrix}$$

So $P \Sigma P'$ is just the scalar $[\Sigma]_{11} = \sigma_0^2$ and

$$\Sigma P' = \begin{pmatrix} \sigma_0^2 \\ \sigma_0^2 \rho \\ \sigma_0^2 \rho \\ \vdots \end{pmatrix}$$

The Black-Litterman market vector mean is

$$\mu_{BL} = \mu + c \Sigma P' (P \Sigma P')^{-1} (v - P \mu) = \begin{pmatrix} (1 - c)\mu_0 + cv_1 \\ (1 - cp)\mu_0 + cpv_1 \\ (1 - cp)\mu_0 + cpv_1 \\ \vdots \end{pmatrix}$$

(1)

Notice that the marginal variances factor out.

To evaluate the $\alpha_{SR}$ portfolio for the second question, we need first to evaluate

$$\Sigma_{BL} = \Sigma - c \Sigma P' (P \Sigma P')^{-1} P \Sigma$$

Since $\Sigma$ is symmetric, $\Sigma P' = (P \Sigma)'$. Thus we can arrive at

$$\Sigma_{BL} = \sigma_0^2 \begin{pmatrix} 1 - c & \rho - cp & \rho - cp & \cdots \\ \rho - cp & 1 - cp^2 & \rho - cp^2 & \cdots \\ \rho - cp & \rho - cp^2 & 1 - cp^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$
The inverse of this is not necessarily apparent, but turns out to be

\[
\Sigma_{BL}^{-1} = \frac{1}{\sigma_0^2 (1 - \rho)} \begin{pmatrix}
\frac{1 - \rho}{1 - c} + \frac{(n-1)\rho^2}{1 + (n-1)\rho} & -\frac{\rho}{1 + (n-1)\rho} & -\frac{\rho}{1 + (n-1)\rho} & \cdots \\
\frac{\rho}{1 - c} - \frac{1 + (n-1)\rho}{1 + (n-1)\rho} & 1 & -\frac{1 + (n-1)\rho}{1 + (n-1)\rho} & \cdots \\
\frac{\rho}{1 - c} - \frac{1 + (n-1)\rho}{1 + (n-1)\rho} & -\frac{1 + (n-1)\rho}{1 + (n-1)\rho} & 1 & \cdots \\
\cdots & \cdots & \cdots & \ddots
\end{pmatrix}
\]  

where \( n = \dim M \) is the number of assets in the investment universe. The proof of this comes from multiplying out \( \Sigma_{BL}^{-1} \) and \( \Sigma_{BL} \).

Assuming the initial price vector \( p \) is one (or at least proportional to one), the SR portfolio allocation is proportional to

\[
\alpha_{SR} \propto \Sigma_{BL}^{-1} \mu_{BL} = \frac{1}{\sigma_0^2} \begin{pmatrix}
v_1 \frac{c}{1-c} + \frac{\mu_0}{1 + (n-1)\rho} \\
\frac{\mu_0}{1 + (n-1)\rho} \\
\frac{\mu_0}{1 + (n-1)\rho} \\
\cdots
\end{pmatrix}
\]

The fraction of the initial value of this portfolio allocated to the first asset is

\[
\left[ \frac{p}{\alpha_{SR}} \right]_1 = \frac{v_1 \frac{c}{1-c} + \frac{\mu_0}{1 + (n-1)\rho}}{v_1 \frac{c}{1-c} + \frac{\mu_0}{1 + (n-1)\rho}}
\]

For a sufficiently broad investment universe, this limits to

\[
\lim_{n \to \infty} \left[ \frac{p}{\alpha_{SR}} \right]_1 = \frac{1}{1 + \frac{c}{v_1 \rho} \frac{1 - c}{1 - c} + \frac{\mu_0}{1 + (n-1)\rho}}
\]  

(3)