Consider a broad investment universe and a normal market vector. Assume all initial asset prices are one. Under the objective probability measure, each pair-wise correlation is \(0 < \rho < 1\) and each component’s mean and variance is \(\mu_0\) and \(\sigma_0^2\) respectively.

The manager’s subjective probability measure is based on a view with confidence \(0 < c < 1\) that one particular market vector component will turn out to be \(v_1 \in \mathbb{R}\).

The picks matrix is simply
\[
P = \begin{pmatrix} 1 & 0 & 0 & \cdots \end{pmatrix}
\]
So \(P\Sigma P'\) is just the scalar \(\Sigma_{11} = \sigma_0^2\) and
\[
\Sigma P' = \begin{pmatrix} \sigma_0^2 \\ \sigma_0^2 \rho \\ \sigma_0^2 \rho \\ \vdots \end{pmatrix}
\]

The Black-Litterman market vector mean is
\[
\mu_{BL} = \mu + c\Sigma P' (P\Sigma P')^{-1} (v - P\mu) = \begin{pmatrix} (1 - c)\mu_0 + cv_1 \\ (1 - c\rho)\mu_0 + c\rho v_1 \\ \vdots \end{pmatrix}
\]
(1)

Notice that the marginal variances factor out.

To evaluate the \(\alpha_{SR}\) portfolio for the second question, we need first to evaluate
\[
\Sigma_{BL} = \Sigma - c\Sigma P' (P\Sigma P')^{-1} P\Sigma
\]

Since \(\Sigma\) is symmetric, \(\Sigma P' = (P\Sigma)'\). Thus we can arrive at
\[
\Sigma_{BL} = \sigma_0^2 \begin{pmatrix} 1 - c & \rho - c\rho & \rho - c\rho & \cdots \\ \rho - c\rho & 1 - c\rho^2 & \rho - c\rho^2 & \cdots \\ \rho - c\rho & \rho - c\rho^2 & 1 - c\rho^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}
\]
The inverse of this is not necessarily apparent, but turns out to be

\[
\Sigma_{BL}^{-1} = \frac{1}{\sigma_0^2(1 - \rho)} \begin{pmatrix}
\frac{1 - \rho}{1 - c} + \frac{(n-1)^2 \rho^2}{1 + (n-1)\rho} & -\frac{\rho}{1 - c} + \frac{(n-1)\rho}{1 + (n-1)\rho} & \cdots \\
-\frac{\rho}{1 - c} + \frac{(n-1)\rho}{1 + (n-1)\rho} & 1 - \frac{\rho}{1 + (n-1)\rho} & \cdots \\
\vdots & \vdots & \ddots \\
\frac{1 - \rho}{1 - c} + \frac{(n-1)\rho}{1 + (n-1)\rho} & -\frac{\rho}{1 - c} + \frac{(n-1)\rho}{1 + (n-1)\rho} & 1 - \frac{\rho}{1 + (n-1)\rho}
\end{pmatrix}
\]  

(2)

where \( n = \dim M \) is the number of assets in the investment universe. The proof of this comes from multiplying out \( \Sigma_{BL}^{-1} \) and \( \Sigma_{BL} \).

Assuming the initial price vector \( p \) is one (or at least proportional to one), the SR portfolio allocation is proportional to

\[
\alpha_{SR} \propto \Sigma_{BL}^{-1} \mu_{BL} = \frac{1}{\sigma_0^2} \begin{pmatrix}
v_1 c + \frac{\mu_0}{1 + (n-1)\rho} \\
v_1 c + \frac{\mu_0}{1 + (n-1)\rho} \\
\vdots \\
v_1 c + \frac{\mu_0}{1 + (n-1)\rho}
\end{pmatrix}
\]

The fraction of the initial value of this portfolio allocated to the first asset is

\[
\frac{[p]_1 [\alpha_{SR}]_1}{p' \alpha_{SR}} = \frac{v_1 c + \frac{\mu_0}{1 + (n-1)\rho}}{v_1 c + \frac{\mu_0}{1 + (n-1)\rho}}
\]

For a sufficiently broad investment universe, this limits to

\[
\lim_{n \to \infty} \frac{[p]_1 [\alpha_{SR}]_1}{p' \alpha_{SR}} = \frac{1}{1 + \frac{1 - c}{v_1 \rho}}
\]

(3)