Revenue Functions

Motivation: Clearly \([\text{profit}] = [\text{in}] - [\text{out}]\).
We've done a lot w. optimizing these \([\text{out}]\) functions, which we're called cost functions, \(C(x)\). Now we need to learn about \([\text{in}]\), which we'll call revenue or \(R(x)\).

* Just like in costs, when we put the word "marginal" in front, it means we're taking a derivative. So "marginal cost" is the first derivative of the cost function and "marginal revenue" is the 1st derivative of the revenue function.

* In general, if we sell \(x\) units at some price \(p\), then our revenue will be \(x \cdot p\). That is

\[
R(x) = xp
\]

* Check: If \(R(x) = xp\), then what is the marginal revenue?

Soln: Marginal revenue is the 1st derivative, so

\[
R'(x) = p
\]
We want to maximize profits. In general, to maximize profits, we raise prices and lower costs. Since we're looking at revenue right now, let's assume our costs are going to be fixed. Then to increase profit, I suppose we want to increase our revenue.

That means our umbrellas will be selling for $20 now instead of $5.

But of course there's a catch! If we make things too expensive, no one will buy them and our sales will go down. Roller Coaster Tycoon taught us that.

So we need to strike some sort of middle ground, somewhere where we can maximize the amount of money we're making.

Here's the mathy stuff: \( \text{Profit} = \text{Price per thing sold} \times \text{quantity sold} \)

In general, you'll sell more at a cheaper price and fewer at a higher price.
We call the equation in the previous graph the "demand equation" \( f(x) \) since it takes as input some quantity \( x \) of things sold and outputs the price at which they would have all been sold, \( p \).

We have: \( p = f(x) \).

Then \( R(x) = x \cdot f(x) \).

- Suppose you're given the demand equation for umbrella sales at your theme park and it is

\[
f(x) = (6 - \frac{1}{2} x)
\]

Find the level of production that will maximize revenue.

1. Setup \( R(x) \). Since \( R(x) = xp = x \cdot f(x) \), we have

\[
R(x) = x \cdot (6 - \frac{1}{2} x)
= 6x - \frac{1}{2} x^2
\]

2. Find where \( R(x) \) attains its maximum value by setting marginal revenue equal to 0 and solving for \( x \). We have
\[ R'(x) = 6 - (2x^{1/2})x = 6 - x \]

\[ 0 = 6 - x \]
\[ x = 6. \]

3 Is \( x = 6 \) a min or a max for \( R(x) \)?

Do a sign chart.

\[ \begin{array}{c|c|c}
\text{R'(x) is positive when} & \text{R'(x) is negative when} \\
X < 6 & X > 6 \\
\hline
\end{array} \]

Then \( x = 6 \) is a maximum of \( R(x) \).

4 Plug \( x = 6 \) into \( R(x) \). We get

\[ R(6) = 18. \]

Then the rate of production that yields the maximum revenue is \( X = 6 \) and my total max revenue is $18.
Exercise 2.7.11

Until recently, hamburgers at the city sports arena cost $4 each. The food concessionaire sold ~10,000 hamburgers per game. When the price was raised to $4.40/hamburger, sales dropped to 8,000/game.

(a) Assuming a linear demand curve, find the price of a hamburger that will maximize nightly hamburger sales.

Demand curve: $p = f(x)$

(1) Since $p = f(x)$ is linear, we can use point-slope form to find $f(x)$.

$(10,000, 4), (8,000, 4.4)$

$m = \frac{\Delta y}{\Delta x} = \frac{.4}{2000} = -.0002$

Point slope form: $(p-y_0) = m(x-x_0)$

$(p-4) = -.0002 (x - 10000)$

$p = -.0002x + 2 + 4$

$p = (.0002)x + 6$
1. Set up \( R(x) \). Since \( R(x) = xp \), we have

\[
R(x) = x (-0.0002x + 6) = -0.0002x^2 + 6x
\]

2. Find extrema of \( R(x) \).

\[
R'(x) = -0.0004x + 6
\]

0 = -0.0004x + 6

-0.0004x = 6

x = 15,000

3. Test to see if \( x = 3 \) is a max or a min.

\[
\begin{align*}
& R'(x) \text{ is positive} & & R'(x) \text{ is negative} \\
& x = 15,000 & & x > 15,000
\end{align*}
\]

It is a max.

4. Find the max price if we were to sell 15,000 burgers.

\[
p = f(15000) = 3
\]

So $3.00/burger will maximize revenue.
(b) If the concessionaire has a fixed cost of $1000/night and $600/hamburger, what hamburger price will maximize nightly profits.

\[ C(x) = 0.6x + 1000 \]

\[ P(x) = R(x) - C(x) \]
\[ = -0.0002x^2 + 6x - 0.6x - 1000 \]
\[ = -0.0002x^2 + 5.4x - 1000 \]

\[ D(x) = -0.0004x + 5.4 \]

\[ 0 = -0.0004x + 5.4 \]
\[ 0.0004x = 5.4 \]
\[ x = 13,500 \]

\[ p = f(13500) = 3.3 \]

So we should sell our hamburgers for $3.30 a piece.