Student Question

**Question.** Why does the distance formula between a point and a plane work?

**Answer.** Maybe the easiest way to explain why this formula works is to show how we derive it. So let’s do that. The whole idea is to get the setup right. Once you create the correct triangle set-up, you can just take one dot product and it gives you the result you need.

Let’s consider a point $R = (x_1, y_1, z_1)$ and a plane $P$ whose points satisfy the equation

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$$  

You should be able to “pluck out” the normal vector to $P$ from this equation. It is $(a, b, c)$. Normalizing the normal vector (that is, making the normal vector have length 1) gives the following vector

$$n = \frac{(a, b, c)}{\|(a, b, c)\|} = \frac{(a, b, c)}{\sqrt{a^2 + b^2 + c^2}}.$$  

Now we have a unit normal vector to the plane, which we can use to project $R = (x_1, y_1, z_1)$ onto $P$. Let’s say that the orthogonal projection of $R$ onto $P$ is the point $Q$. Then the direction vector between $R$ and $Q$ is some scalar multiple of $n$.

Additionally, here is a vector $v$ that goes between the base point of the plane (which you can read off from the formula, it is: $(x_0, y_0, z_0)$). Since $v$ is just the directional vector between the base point of $P$ and $R$ so we can compute it. It is

$$v = R - (x_0, y_0, z_0) = (x_1 - x_0, y_1 - y_0, z_1 - z_0).$$  

We have created a nice little triangle for ourselves. There is a picture on p.43 of your text book (although my letters are slightly different than the book’s notation). The punchline is: if we can project $v$ onto $n$, then we will have the length of the vector between $R$ and $Q$. We can do that by taking $|v \cdot n|$. That gives the distance formula as required.