Parameterizations of Surfaces

1. Find a parameterization of the surface $z = 3x^2 + 8xy$ and use that parameterization to find the tangent plane at $x = 1, y = 0, z = 3$.

2. Consider the following parameterization of an ellipsoid

\[
\phi(u, v) = \begin{pmatrix}
    a \sin u \cos v \\
    b \sin u \cos v \\
    c \cos u
\end{pmatrix}
\]

for $b < a$ and $0 \leq u \leq 2\pi$ and $0 \leq v \leq 2\pi$. Verify that all the points in the image of $\phi$ satisfy the Cartesian equation of an ellipsoid:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.
\]

3. Determine if the parameterization in Question 2 is regular.

4. If the parameterization in Question 2 is regular, set up (but do not evaluate) the integral for the surface area $A(\text{ellipsoid})$. 

\[
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\]
Surface Area

5. Let \( \phi(u, v) = (e^u \cos v, e^u \sin v, v) \) for \( u \in [0, 1] \) and \( v \in [0, \pi] \).

(a) Find \( T_u \times T_v \).

(b) Find the (equation for) the tangent plane to \( S \) when \( (u, v) = (0, \pi) \).

(c) Find the area \( \phi(D) \) where \( D = [0, 1] \times [0, \pi] \).

6. Let \( \phi(u, v) = (u - v, u + v, uv) \) and let \( D \) be the unit disk in the \( uv \)-plane.

Find the area of \( \phi(D) \).