Computational Exercises

1. Find the parametric equations of the line through (1, 2, 3) and parallel to the vector $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

   \[
   \begin{align*}
   x &= 3t + 1 \\
   y &= -2t + 2 \\
   z &= t + 3
   \end{align*}
   \]

2. Write a set of parametric equations for the line with symmetric form

\[
\frac{x+1}{2} = \frac{y-1}{-7} = \frac{z+10}{4}.
\]

   \[
   \begin{align*}
   x &= 2t - 1 \\
   y &= 1 - 7t \\
   z &= 4t - 10
   \end{align*}
   \]

3. Parametrize the line described by $y = 3x + 1$.

   \[
   \begin{align*}
   x &= t \\
   y &= 3t + 1
   \end{align*}
   \]

4. Consider the line defined by $f(t) = (t, 3t + 1, -4t)$. Does this intersect with the plane $x + y + z = 1$?

   \[
   x + y + z = t + (3t + 1) + (-4t) = t + 3t - 4t + 1 = 1
   \]

   so the answer is yes.
Vicki the Vulture Flies to Vincent Hall

5. Vicki the Vulture is flying over the Mississippi with trajectory given by parametric equations

\[ f(t) = \begin{cases} 
  t + 1 \\ 2t - 2 \\ 20 
\end{cases} \]

At the same time, a flock of butterflies is flying in a formation shaped like

\[ (t + 1, 2s - 7 + 3t, 20) \]

If Vicki does not change course, will she fly through the butterflies? If so, when?

This is a question of the intersection between a line and a plane. In this case its easy because setting \( s = \frac{5 - t}{2} \) gives \( y = 2(\frac{5 - t}{2}) - 7 + 3t = 2t - 2 \) on the plane. So Vicki is flying with the butterflies.

6. A student is standing on the bridge. If the student is at position \((4, 4, 10)\). When does Vicki pass the student? What is Vicki's position when she passes the student?

Q: When does Vicki intersect the plane \((4, 4, 5)\)?

A: When \( t + 1 = 4 \), \( 2t - 2 = r \), and \( 20 = s \).

Note: \( t + 1 = 4 \) forces \( t = 3 \). So \( 2(3) - 2 = 4 = r \)

Thus, Vicki passes directly over the student when \( t = 3 \); \((4, 4, 20)\)

7. Vicki spots a left over sandwich on the quad in front of Vincent Hall. Given that she is 20 feet above the ground and circles down in a circle of radius of 5, model her descent using parametric equations.

There are many solutions. Here is one:

\[ x = 5 \cos t \]
\[ y = 5 \sin t \]
\[ z = 20 - t \]