Computational Exercises

Let \( a = (1, 2, 3) \) and \( b = (1, 0, 4) \).

1. What is \( a \cdot b \)?

\[
(1, 2, 3) \cdot (1, 0, 4) = 1 + 0 + 12 = 13
\]

2. What is the angle between \( a \) and \( b \)?

\[
\|a\| \cdot \|b\| \cdot \cos \Theta = \sqrt{14} \cdot \sqrt{17} \cdot \cos \Theta
\]

\[
\Rightarrow \Theta = \cos^{-1} \left( \frac{13}{\sqrt{238}} \right)
\]

3. What is \( \text{proj}_a b \)?

\[
\left( \frac{a \cdot b}{a \cdot a} \right) a = \left( \frac{13}{14} \right) \cdot (1, 2, 3) = \left( \frac{13}{14}, \frac{13}{7}, \frac{13}{14} \right)
\]

4. Give the unit vector in the same direction as \( b \).

\[
\frac{b}{\|b\|} = \left( \frac{1}{\sqrt{17}}, 0, \frac{4}{\sqrt{17}} \right)
\]

5. Compute \( a \times b \). Compare this \( b \times a \).

\[
a \times b = (2, -1, -2)
\]

\[
b \times a = (-2, 1, 2)
\]

6. What is \( i \times i \)? What is \( i \times j \)? (Can you guess what \( i \times k \) will be?)

\[
i \times i = \mathbf{0}
\]

\[
i \times j = k
\]

\[
i \times k = j
\]
Moving Day for Vicki the Vulture

7. Vicki pushes her (50 lb) couch up a 4 ft hill to her front door. If the hill is at a 30° angle from the horizontal, how much work does she do?

\[
\text{Work} = F \cdot \text{displacement} = \| F \| \cdot \| \text{displacement} \| \cdot \cos \Theta
\]

\[
= 50 \cdot 4 \cdot \cos(30°)
\]

8. Vicki drags her fridge 12 ft across her (smooth) floor, using a rope and 60 lbs of force directed alongside the rope. How much work is done if the rope makes a 20° angle with the horizontal?

\[
\text{Work} = F \cdot \text{displacement} = \| F \| \cdot \| \text{displacement} \| \cdot \cos \Theta
\]

\[
= 60 \cdot 12 \cdot \cos 20°
\]

\[
\approx 293.819
\]

9. Vicki's bedroom has an unusual shape. The corners of the bedroom are at (1, 1, 0), (3, 2, 0), and (1, 3, 0), and (−1, 2, 0). How much floorspace does she have?

Area:

\[
\| \vec{AB} \times \vec{AC} \| = \| (2,1,0) \times (2,-1,0) \|
\]

\[
= \| (0,0,-4) \| = 4
\]

10. Vicki's patio has an unusual shape. The corners of the patio are at (1, 2, 3), (4, −2, 1), and (−3, 1, 0), and (0, −3, 2). How much floorspace does she have?

\[
AB = (3, -4, -2) \quad \text{and} \quad AC = (-4, -1, -3)
\]

\[
\vec{AB} \times \vec{AC} = (10, 12, 19)
\]

\[
\| (10, 12, 19) \| = 5\sqrt{30}
\]