Computational Exercises

1. Let \( f(x, y) = x^2 + 2y \).

   (a) What is \( \frac{\partial f}{\partial x} \)?

   (b) What is \( \frac{\partial f}{\partial y} \)?

   (c) What is \( \frac{\partial}{\partial x} \cdot \frac{\partial f}{\partial y} \)?

   (d) Is this the same as \( \frac{\partial}{\partial y} \cdot \frac{\partial f}{\partial x} \)?

2. Let \( g(t) = e^{x^2-y^2} \).

   (a) What is \( \frac{\partial g}{\partial x} \)?

   (b) What is \( \frac{\partial g}{\partial y} \)?

   (c) What is \( \frac{\partial}{\partial x} \cdot \frac{\partial g}{\partial x} \)? What is \( \frac{\partial}{\partial y} \cdot \frac{\partial g}{\partial y} \)?

3. Determine all partial derivatives (including mixed partials) of \( h(t) = \sin(\ xy) \).

4. Determine all partial derivatives (including mixed partials) of \( f(t) = \ln(\frac{y}{x}) \).
Modelling Heat

5. Consider a straight wire along the $x$ axis. The temperature $T(t, x)$ is modelled by an equation satisfying

$$k \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}.$$

Show that $T(x, t) = e^{-kt} \cos x$ could be a model for the temperature of the wire.

6. In two dimensions, the heat equation can model heat on a plate. It states

$$k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = \frac{\partial T}{\partial t}.$$

Graph the surfaces given by $z = T(x, y, 1)$, $z = T(x, y, 5)$, and $z = T(x, y, 10)$.

7. In three dimensions, the heat equation states

$$k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = \frac{\partial T}{\partial t}.$$

Show that $T(x, y, z, t) = e^{-kt} (\cos x + \cos y + \cos z)$ satisfies the three-dimensional heat equation.