Directional Derivatives

All the problems on this side are about $f(x, y) = x^2 + x \sin y$.

1. Compute $\nabla f(x, y)$.

2. For $\vec{a} = (1, \pi)$ and $v = (1, 0)$, compute $D_v f(\vec{a})$.

3. Suppose $\vec{a} = (0, 0)$. Compute $D_v f(\vec{a})$ for arbitrary $v$.

4. For $\vec{a} = (1, \pi)$, find a $v$ for which compute $D_v f(\vec{a})$ is maximized. What is that maximum? Find a vector that minimizes $D_v f(\vec{a})$.

5. Suppose $\vec{a} = (0, 0)$. Find a $v$ for which compute $D_v f(\vec{a})$ is minimized. What is that minimum?
Tangent (Hyper)Planes

6. Let $f(x, y, z) = 2x - y + 5z$. Find the tangent plane to $f$ at the point $\vec{a} = (1, 2, 3)$ using the gradient of $f$.

7. The sphere of radius 1 is described by $(1, \phi, \theta)$ where $0 \leq \phi \leq \pi$ and $0 \leq \theta \leq 2\pi$. Give an equation for a plane tangent to the sphere at the point $(\rho, \phi, \theta) = (1, \frac{\pi}{4}, 0)$.

8. Let $g(x, y, z, w) = xyzw + y^2zw + z^3w$. Find the tangent hyperplane to this surface at the point $\vec{b} = (1, 0, -1, 1)$. 