Hiking!

Vicki is hiking along a series of hills and valleys. Her current height is described by the equation

\[ h(x, y) = \sin \frac{x}{2} \cdot \cos 2y \]

where \( x \) indicates her east/west position and \( y \) indicates her north/south position.

1. She is currently at latitude \( 3\pi \) and longitude \( \frac{\pi}{2} \), what is her height?

2. Is Vicki currently at the peak of a hill? Why or why not? (Explain your reasoning using the gradient of \( h \).)

3. Give a linear approximation of the area near Vicki’s current location.

4. What direction should she walk in order to go downhill the fastest? How much does her height decrease if she walks in that direction for 1 minute?
Divergence at a point in a vector field quantifies how much the vector field is flowing away from you.

5. Let \( F(x, y) = \begin{pmatrix} \sin x \\ \cos y \\ z^2 \end{pmatrix} \). Compute the divergence at \( (0, \frac{\pi}{2}, 0) \) and \( (\frac{\pi}{2}, \frac{\pi}{2}, 0) \). Is the vector field moving toward or away from those two points?

Curl at a point in a vector field tells you the speed and direction of motion.

6. Let \( G(x, y) = \begin{pmatrix} \ln y \\ \sin x \\ e^z \end{pmatrix} \). Compute the curl at \( (0, 1, 1) \) and \( (\pi, 1, 1) \). Is the direction of the vector field different at those two points? What about the speed?

Looking at curl and divergence together can help you understand a vector field at a point.

7. Let \( H(x, y) = \begin{pmatrix} e^x \\ y^2 + x \\ \sin x \end{pmatrix} \). Describe the motion of the vector field at \( (1, \pi, 1) \) and \( (1, \frac{\pi}{2}, 0) \). Is the motion of the vector field different at those two points? How?