Formal Properties of Integrals

1. Linearity of integration. Evaluate $\int \int_D x^2 y - 3y \, dA$ and $\int \int_D x^2 y \, dA - 3 \int \int 3y \, dA$ where $D$ is the region defined by $D = \{(x, y) \mid 0 \leq x \leq 1, \sqrt{x} \leq y \leq x^3\}$. How are these two values related?

2. Integrals satisfy a version of the triangle inequality. Let $f(x, y) = -4y^3x - 3x^2$. Evaluate $\int \int_R |f| \, dA$ and $|\int \int_R f \, dA|$ where $R = \{(x, y) \mid -1 \leq x \leq 1, -2 \leq y \leq 1\}$. How are these two values related?

3. Integration of dominating functions. Let $f(x, y) = x^3 + x^2y + xy^2 + 3y^3$ and $g(x, y) = x^2y + 2y^3$. Compare $\int \int \left| f(x, y) \right| \, dA$ and $\int \int \left| g(x, y) \right| \, dA$ over the square $\{(x, y) \mid 0 \leq x \leq 3, 0 \leq y \leq 3\}$. 


Exam II Review

4. Let $F(x, y) = (xy^2, x^2y, x^3 + y)$ and $g(t) = (\sin t, e^t)$. Compute $D(F \circ g)$. (2.5)

5. Could $F(x, y) = (xy^2, x^2y, x^3 + y)$ be a gradient vector field? Why or why not? (3.3)

6. Let $f(x, y, z) = (x^3 + 3y^3 + 2z^3 + 2xy + zy + 3xz)$. Compute the curl and divergence of the vector field $\nabla f$. (2.6), (3.4)

7. Find an equation for the tangent plane to

$$x^3 + y^2 + z^3 = 7$$

at the point $(x_0, y_0, z_0) = (0, -1, 2)$ in two ways: (1) using the Taylor expansion and (2) using the level-curve method. (2.6), (4.1)
The following survey asks for feedback about your TA.

1. I learn best from (circle all that apply)...
   ... seeing examples at the board
   ... doing examples by myself
   ... teaching others
   ... other (please elaborate):

2. One thing I like about discussion:

3. One thing I’d like to see more of in discussion:

4. Other comments?