1. Look at all of the following theorems and equations. For each item, Identify the equation and give its name (if it is a theorem). Recall the situation in which the equation is used.

<table>
<thead>
<tr>
<th>A</th>
<th>[ \iint_X F dS = \iint_R F(f(s,t)) \cdot \mathbf{N}(s,t) dsdt ]</th>
<th>Surface Integral of a vector function</th>
</tr>
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<tbody>
<tr>
<td>B</td>
<td>[ \nabla \times F ]</td>
<td>Curl of function</td>
</tr>
<tr>
<td>C</td>
<td>[ \oint_{\partial M} F dx = \iint_R \text{curl}(F) ]</td>
<td>Stokes’s Theorem</td>
</tr>
<tr>
<td>D</td>
<td>[ \iint_S g(x,y,z) dxdydz = \iint_{S^*} g(f(s,t,u))</td>
<td>\text{det}(J)</td>
</tr>
<tr>
<td>E</td>
<td>[ x = \rho \sin(\varphi) \cos(\theta) ] [ y = \rho \sin(\varphi) \sin(\theta) ] [ z = \rho \cos(\varphi) ] [ dxdydz = \rho^2 \sin(\varphi) d\rho d\theta d\varphi ]</td>
<td>Spherical Coordinates</td>
</tr>
<tr>
<td>F</td>
<td>[ x = r \cos(\theta) ] [ y = r \sin(\theta) ] [ z = w ] [ dxdydz = rdrd\theta dw ]</td>
<td>Cylindrical Coordinates</td>
</tr>
<tr>
<td>G</td>
<td>[ A = \iint_S 1 dS ]</td>
<td>Surface Area</td>
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<tr>
<td>H</td>
<td>[ \nabla \cdot F ]</td>
<td>Divergence of a function</td>
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</table>

- Calculate
  \[ \iint_S (x^2 + 2z \sin(y)) dxdydz \]

  Where \( S \) is the solid parameterized by \( f(r,s,\theta) = (r \sin(\theta), 2r \cos(\theta), 2s + r) \)

- Calculate the surface integral of \( F = xz, 2x - z, 3xy \) on a parameterized potato chip.

- Determine if a fluid with velocity \( F = (xz, 2x - z, 3xy) \) is compressible.

- Find the line integral of \( F = (xz, 3x - z, 3xy) \) on the intersection of the saddle surface \( z = x^2 - y^2 \) and the cylinder \( x^2 + y^2 = 9 \)

- Evaluate \[ \int_{-2}^{2} \int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}} 2z \sin(y) dxdy \]

- Evaluate \[ \int_{-2}^{2} \int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}} x^2 + 2z \sin(y) dxdy \]

- Describe the rotation of \( F = (xy, 2x - z, 3xy) \) at the point \( (1,1,1) \)

- Calculate the surface integral of \( F(x,y,z) = xz - 2y \) on the surface of a parameterized pringle chip.

- Calculate the surface area of the cone \( z^2 = x^2 + y^2 \)

  If you are using the correct formulas you should end up spelling the word “mathrocks”
Your book defines the first and second order Taylor polynomials on page 196. Using more compact notation, we can write the Taylor Formula about the point \( x_0 \) as

\[
f(x) = f(x_0) + Df(x_0)(x - x_0) + x^t H f(x_0) x
\]

2. Write out Taylor’s Formula explicitly for scalar valued functions of one, two, and three variables.

In dimension 1 this is the same Taylor polynomial that we worked with in single variable calculus. We will let our point \( a = a \).

\[
p_1(x) = f(a) + f'(a)(x - a)
\]

\[
p_2(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2} (x - a)^2
\]

Now for \( n = 2 \) the two dimensional case, rather than using \( x_1 \) and \( x_2 \) as variables we will use \( x \) and \( y \), and we will write \( a = (a, b) \).

\[
p_1(x, y) = f(a) + f_x(a)(x - a) + f_y(a)(y - b)
\]

\[
p_2(x, y) = f(a) + f_x(a)(x - a) + f_y(a)(y - b) + \frac{f_{xx}(a)}{2} (x - a)^2 + \frac{f_{yy}(a)}{2} (y - b)^2 + f_{xy}(a)(x - a)(y - b)
\]

Finally if \( n = 3 \), the 3 dimensional case, we will use \( x, y, z \) for our variables, and let the point \( a = (a, b, c) \) we have

\[
p_1(x, y, z) = f(a) + f_x(a)(x - a) + f_y(a)(y - b) + f_z(a)(z - c)
\]

\[
p_2(x, y, z) = f(a) + f_x(a)(x - a) + f_y(a)(y - b) + f_z(a)(z - c) + \frac{f_{xx}(a)}{2} (x - a)^2 + \frac{f_{yy}(a)}{2} (y - b)^2 + \frac{f_{yy}(a)}{2} (z - c)^2 + f_{xy}(a)(x - a)(y - b) + f_{xz}(a)(x - a)(z - c) + f_{yz}(a)(y - b)(z - c)
\]

3. Consider the function \( f(x, y) = \sin(xy) \) and

- Compute the Hessian matrix of \( f \) at the point \( a = (0, 0) \).

The Hessian matrix in this situation will be given as

\[
H = \begin{pmatrix}
\frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\
\frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2}
\end{pmatrix}
\]

To begin we will compute

\[
\frac{\partial f}{\partial x} = f_x(x, y) = y \cos(xy)
\]

\[
\frac{\partial f}{\partial y} = f_y(x, y) = x \cos(xy)
\]
Building off of this result, we can easily compute the whole Hessian

Which we can easily compute explicitly

\[
H = \begin{pmatrix}
-y^2 \sin(xy) & \cos(xy) - xy \sin(xy) \\
\cos(xy) - xy \sin(xy) & -x^2 \sin(xy)
\end{pmatrix}
\]

- Compute the degree 2 Taylor Polynomial of \( f(x, y) \) at the point \( a = (0, 0) \)
  Using the general form we compute above, and the Hessian matrix (which we also computed)

\[
p_2(x, y) = \sin(0 \cdot 0) + 0 \cdot \cos(0 \cdot 0)(x - 0) + 0 \cdot \cos(0 \cdot 0)(y - 0) - 0^2 \sin(0 \cdot 0) - 0^2 \sin(0 \cdot 0) + \cos(0 \cdot 0) - 0 \cdot 0 \sin(x \cdot y) + \cos(0 \cdot 0)
\]

4. Let \( \mathbf{F}(x, y, z) = (xy^2, yx^2, -z^2) \), use the divergence theorem to calculate the total flux out of a canister \( W \), which can be expressed by the equations \( x^2 + y^2 \leq 9 \) and \( 1 \leq z \leq 3 \).

We will solve this problem using the divergence theorem. As the problem is stated we want compute a flux (surface) integral over the boundary of \( W \). That is we want to compute

\[
\iint_{\partial W} \mathbf{F} \, dS
\]

Using the divergence theorem this becomes

\[
\iiint_{W} \nabla \cdot \mathbf{F} \, dA
\]

It is not difficult to compute the div of \( \mathbf{F} \)

\[
\nabla \cdot \mathbf{F} = \frac{\partial}{\partial x} (xy^2) + \frac{\partial}{\partial y} (yx^2) + \frac{\partial}{\partial z} (-z^2) = y^2 + x^2 - 2z
\]
So we setup the integral according to the divergence theorem, and writing the region in cylindrical coordinates

\[ \iiint_{W} (y^2 + z^2 - 2z) = \int_{1}^{3} \int_{0}^{2\pi} \int_{0}^{r^2 - 2z} r \, dr \, d\theta \, dz = 9\pi \]