1. Let \( S \) denote the closed cylinder with bottom given by \( z = 0 \) and top given by \( z = 4 \) and the lateral surface given by \( x^2 + y^2 = 9 \). Orient \( S \) with outward normals. Determine the Surface Integral

\[
\iint_S (x\mathbf{i} + y\mathbf{j})dS
\]

a) Is this a vector or a scalar surface integral?
b) Over what surface are we integrating? Can you write it with parametric equations?
c) Find a outward facing normal vector
d) Compute the integral

2. Let \( C \) be the boundary of the surface \( z = x^2 + y^2 \) with \( 0 \leq x \leq 2 \) and \( 0 \leq y \leq 1 \), oriented with upward facing normal. Define \( F(x, y, z) = (\sin(x^3) + xz, x - yz, \cos(z^4)) \) and evaluate

\[
\int_C F \cdot ds
\]
3. The helicoid surface $S$ is parameterized by $\mathbf{X}(s, t) = (s \cos(t), s \sin(t), t)$ for $0 \leq s \leq 1$ and $0 \leq t \leq \pi/2$. Compute the line integral

$$\oint_{\partial S} \mathbf{F} \cdot d\mathbf{s}$$

For the function $\mathbf{F}(x, y, z) = zi + xj + yk$

4. Let $S$ be the hemisphere $x^2 + y^2 + z^2 = 4$ with $z \geq 0$ oriented upwards. Let $\mathbf{F}(x, y, z) = (x^2e^{yz}, y^2e^{xz}, z^2e^{xy})$ be a vector field. Evaluate:

$$\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$$

5. Let $\mathbf{F}(x, y, z) = (xy, e^{z^2} + y, x + y)$ and let $S$ be the graph of the function $y = x^2/9 + z^2/9 - 1$ with $y \leq 0$ oriented so that the normal vector $S$ has positive $y$ component. Compute the integral

$$\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$$