1. Evaluate the triple integral.

\[\int_{-1}^{2} \int_{1}^{0} \int_{-1}^{1} 3yz^2 dxdydz\]

(work coming soon)

The end value of this integral is

\[\frac{1539}{16} \approx 96.1875\]

2. Setup and evaluate the integral

\[\iiint_{V} xyz dV\]

Where \(V\) is the region bounded by a cube, each side of which has length 2 centered and is centered at \((0, 1, 2)\).

The correct way to setup this integral is

\[\int_{-1}^{1} \int_{0}^{3} \int_{1}^{2} xyz dz dy dx\]

After computing this integral we find the value of the integral to be 0
(work coming soon)

3. Consider but do Not evaluate the integral

\[\int_{-2}^{2} \int_{0}^{\sqrt{4-x^2}} \int_{0}^{\sqrt{4-x^2-y^2}} 2dz dy dz\]

(a) What is the region of integration?

The region of integration is the section of the sphere of radius 2 given by \(x^2+y^2+z^2=4\) with \(z \geq 0\). In particular it is a hemisphere centered at \((0, 0, 0)\) located entirely above the \(xy\) plane.
(b) Can you find the value of this integral using elementary means? This integral we can compute without any knowledge of calculus. The value of the integral will be twice the volume of a hemisphere. The area of a hemisphere will be half the volume of a sphere. The volume of a sphere of radius $r$ is given by

$$V = \frac{4}{3} \pi r^3$$

So the value of the integral will be

$$2 \cdot \frac{1}{2} \cdot \frac{4}{3} \pi \frac{8}{3} = \frac{32 \pi}{3}$$

(c) Write the bounds for this integral in spherical coordinates

In spherical coordinates the bounds for a hemisphere will be

$$0 \leq \rho \leq 2$$
$$0 \leq \theta \leq 2\pi$$
$$0 \leq \phi \leq \frac{\pi}{2}$$

One might then try and compute the integral using these bounds.

$$\int_0^2 \int_0^{2\pi} \int_0^{\frac{\pi}{2}} 2 \rho \phi \, d\phi \, d\theta \, d\rho = 4\pi^2$$

Which is not the same answer. It so happens that when one changes coordinate systems one must also adjust the integration, so we can no longer simply integrate $dxdydz$ we must now integrate $\rho^2 \sin \phi \, d\rho \, d\theta \, d\phi$

4. Decide whether or not the following vector fields are conservative. If they are conservative, find a potential function.

(a) $(yz, xz, xy)$

To begin we check if this vector field is conservative, using the test that a vector field is conservative if and only if $\text{curl } F = 0$. 


\[ \text{curl}\, F = \det \begin{pmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{pmatrix} = i \left( \frac{\partial}{\partial y} xy - \frac{\partial}{\partial z} zx \right) + j \left( \frac{\partial}{\partial x} xy - \frac{\partial}{\partial z} yz \right) - k \left( \frac{\partial}{\partial x} xz - \frac{\partial}{\partial y} xz \right) = i(x - x) - j(y - y) + j(z - z) = (0, 0, 0) \]

We can guess a potential function

\[ f(x, y, z) = xyz \]

And check that it really is a potential function

\[ \nabla f = (yz, xz, xy) \]

(b) \((xy^2z^3, 2x^2yz^3, 3x^2y^2z^2)\)

To begin we check if this vector field is conservative, using the test that a vector field is conservative if and only if curl \( F = 0 \).

\[ \text{curl}\, F = \det \begin{pmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yze^{xz} & e^{xz} & xye^{xz} \end{pmatrix} = i(6x^2yz^2) - j(-3xy^2z^2) + j(4xyz^2 - 2xyz^3) \neq (0, 0, 0) \]

So this vector field is not conservative.

(c) \((yze^{xz}, e^{xz}, xye^{xz})\)

To begin we check if this vector field is conservative, using the test that a vector field is conservative if and only if curl \( F = 0 \).

\[ \text{curl}\, F = \det \begin{pmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yze^{xz} & e^{xz} & xye^{xz} \end{pmatrix} = i \left( \frac{\partial}{\partial y} xye^{xz} - \frac{\partial}{\partial z} e^{xz} \right) + j \left( \frac{\partial}{\partial x} xye^{xz} - \frac{\partial}{\partial z} yze^{xz} \right) - k \left( \frac{\partial}{\partial x} e^{xz} - \frac{\partial}{\partial y} yze^{xz} \right) = (0, 0, 0) \]
So this vector field is conservative.
We can guess a potential function

\[ f(x, y, z) = ye^{xz} \]

And check that it really is a potential function

\[ \nabla f = (yz e^{xz}, e^{xz}, xye^{xz}) \]