Exam 3 Review
3.5 - 3.7, 4.4, 5.1 - 5.5

3.5: Transformations of Basic Curves

- Memorize the basic shape of the graphs \( f(x) = x^2, f(x) = x^3, f(x) = x^4, f(x) = |x|, f(x) = \sqrt{x} \)
- Memorize the general properties of transformations that apply to \( f(x) = x^2 \) and to the other basic graphs discussed in this section:
  - \( g(x) = f(x) + k \) is a vertical shift
  - \( g(x) = f(x - h) \) is a horizontal shift
  - \( g(x) = f(-x) \) is a reflection across the y-axis
  - \( g(x) = -f(x) \) is a reflection across the x-axis
  - \( g(x) = af(x) \) is a stretch or scrunch depending on the value of \( a \), for \( a > 0 \)

1. Graph \( f(x) = -2|x+3| \)

**Solution:** The graph of \( f(x) = |x| \) is flipped across the x-axis, y-values double (stretched by a factor of 2), and moved 3 units left

2. Graph \( g(x) = \sqrt{-x} + 2 \)

**Solution:** The graph of \( g(x) = \sqrt{x} \) is reflected across the y-axis and shifted up 2 units

3.6: Combining Functions

- Given \( f(x) \) and \( g(x) \), be able to find \( f + g, f - g, f \cdot g, f/g, f \circ g, \) and \( g \circ f \)
- Be able to determine the domain and range of \((f \circ g)(x)\) and \((g \circ f)(x)\)

3. If \( f(x) = \frac{1}{x} \) and \( g(x) = 4x - 9 \), find \((f \circ g)(x)\) and \((g \circ f)(x)\). Give the domain and range of each.

**Solution:**

\[
(f \circ g)(x) = \frac{1}{4x - 9}, \text{ Domain: } \{x|x \neq 9/4\}, \text{ Range: } \{y|y \neq 0\}
\]

\[
(g \circ f)(x) = \frac{4}{x} - 9, \text{ Domain: } \{x|x \neq 0\}, \text{ Range: } \{y|y \neq -9\}
\]

4. If \( f(x) = \sqrt{x} \) and \( g(x) = 3x - 5 \), find \((f \circ g)(x)\) and \((g \circ f)(x)\). Give the domain and range of each.

**Solution:**

\[
(f \circ g)(x) = \sqrt{3x - 5}, \text{ Domain: } \{x|x \geq 5/3\}, \text{ Range: } \{y|y \geq 0\}
\]

\[
(g \circ f)(x) = 3\sqrt{x} - 5, \text{ Domain: } \{x|x \geq 0\}, \text{ Range: } \{y|y \geq -5\}
\]
3.7: Inverse Functions

• Given $f(x)$, be able to find $f^{-1}(x)$
• Given $f(x)$, be able to determine the domain and range of $f^{-1}(x)$
• Given two functions, determine if they are inverses or not
• Determine whether a function is one-to-one
• Determine where a graph is increasing or decreasing

5. Determine if these two functions are inverses. Support your answer. $f(x) = 3x - 4$, $g(x) = \frac{1}{3}x + 4$

**Solution:** $f(x)$ and $g(x)$ are not inverses because $f(g(x)) \neq x$ and $g(f(x)) \neq x$

6. For the function $f(x) = (x + 3)^2$, $x \geq -3$:
   (a) Is $f(x)$ one-to-one?

   **Solution:** Yes $f(x)$ passes the horizontal line test

   (b) Find the domain and range of $f(x)$.

   **Solution:** Domain: $\{x | x \geq -3\}$, Range: $\{y | y \geq 0\}$

   (c) Find the domain and range of $f^{-1}(x)$.

   **Solution:** Domain: $\{x | x \geq 0\}$, Range: $\{y | y \geq -3\}$

   (d) Find $f^{-1}(x)$.

   **Solution:** $f^{-1}(x) = \sqrt{x} - 3$

   (e) Determine where $f(x)$ and $f^{-1}(x)$ are increasing and decreasing.

   **Solution:** $f(x)$ is increasing for $x \geq -3$, $f^{-1}(x)$ is increasing for $x \geq 0$
4.4: Graphing Polynomials

- Be able to factor a quadratic function
- Be able to test points to determine where a function is positive or negative

7. Graph \( f(x) = x^4 + 12x^3 + 35x^2 \). Label the \( x \)-intercepts, \( y \)-intercept, and 4 additional points (one in each interval).

**Solution:** The \( x \)-intercepts are \((-7,0), (-5,0), \) and \((0,0)\). Four additional points are \((-8,192), (-6,-36), (-1,24), \) and \((1,48)\).

5.1: Exponents and Exponential Functions

- Memorize the exponent rules in Property 5.1 (p. 390)
- Memorize the general shape of the exponential function \( f(x) = b^x \). Be able to find the \( y \)-intercept of this function.
- Be able to plot functions similar to \( f(x) = b^x \). You can do this either by using the transformation rules that apply to parabolas or by plotting points.
- Be able to solve equations with variables as exponents

8. Graph \( f(x) = -2^x \)

**Solution:** The graph of \( f(x) = 2^x \) is reflected across the \( x \)-axis

9. Graph \( f(x) = 2^{-x} \)

**Solution:** The graph of \( f(x) = 2^x \) is reflected across the \( y \)-axis

10. Solve for \( x \): \( 9^{x-5} = 27^{2x+1} \)

**Solution:** \( 3^{2x-10} = 3^{6x+3} \) yields \( x = -13/4 \)
5.2: Applications of Exponential Functions

- Formulas for annually, periodically, and continuously compounded interest will be given. The formula for continuously compounded interest can be used to model anything with exponential growth, such as a population. You must know what the letters mean and when to use the formulas.

\[ A = P(1 + r)^t \quad A = P \left( 1 + \frac{r}{n} \right)^{nt} \quad A = Pe^{rt} \]

11. You invest $2,000 in an account where interest is compounded 3 times each year. What is the interest rate if the investment grows to $5,000 in 6 years? Round to the nearest tenth of a percent.

Solution: We use the formula \( 5,000 = 2,000 \left( 1 + \frac{r}{3} \right)^{3\cdot6} \) to determine that \( r \approx 15.7\% \)

12. A certain radioactive substance decays according to the model \( Q_t = Q_0 \left( \frac{1}{2} \right)^{t/4,000} \), where \( Q_0 \) is the initial amount of the substance and \( t \) is times measured in years. If we begin with 6 grams of that substance, how much will remain in 500 years? Round to the nearest milligram.

Solution: We use the formula \( Q_{500} = 6 \left( \frac{1}{2} \right)^{500/4,000} \) to determine that approximately 5.502 grams will remain after 500 years.

13. The population of an ant colony is given by \( Q_t = Q_0e^{0.67t} \), where \( t \) is measured in months. If there are 50 ants in the colony right now, how many ants will be in the colony in 1 year?

Solution: We use the formula \( Q_{12} = 50e^{0.67\cdot12} \) to determine that there will be 155,131 ants in one year.
5.3: Logarithms

- Memorize the equivalence $\log_b r = t \iff b^t = r$
- Memorize Properties 5.5, 5.6, and 5.7 and be able to use them to solve problems

14. Write as a sum or difference of simpler logarithms. $\log_5 \sqrt[3]{x^3 y^7}$

**Solution:** $\frac{3}{5} \log_5 (x) + \frac{7}{5} (y)$

15. Evaluate $\log_5 \left( \frac{\sqrt{125}}{25} \right)$

**Solution:** Rewrite everything with base 5 to find $-1/2$

16. Rewrite $2 \log_5 (x) - 4 \log_5 (xy)$ as a single logarithm.

**Solution:** $\log_5 \left( \frac{1}{x^2 y^4} \right)$

5.4: Logarithmic Functions

- Be able to graph $f(x) = \log_{10}(x)$ and $f(x) = \ln(x)$

17. Graph $f(x) = \log_{10}(x) - 5$

**Solution:** The basic graph of $f(x) = \log_{10}(x)$ is shifted down 5 units.

18. Find $x$ and $y$ in these equations: $\log x = 3.456$ and $\ln y = 5/4$. Approximate to 2 decimal places.

**Solution:** $x \approx 2857.59$, $y \approx 3.49$


5.5: Exponential and Logarithmic Equations

- Memorize Property 5.9
- Be able to solve equations with a variable as an exponent

19. Solve $4^{x+3} = 15$. Approximate the solution to four decimal places.

**Solution:** $x \approx -1.0466$

20. Solve $\log x + \log(x - 9) = 1$

**Solution:** $x = 10$ and $x = -1$ but we must exclude $x = -1$ since we can’t take the logarithm of a negative number. So, $x = 10$ is the only solution.

21. Find $\log_7 84$. Approximate the solution to three decimal places.

**Solution:** $\log_7 84 \approx 2.277$

22. $2,000$ is invested in an account with $6\%$ interest compounded monthly (12 times per year). How many years will it take for the investment to grow to be $5,000? Round to the nearest tenth of a year.

**Solution:** We use the formula $5,000 = 2,000 \left(1 + \frac{0.06}{12}\right)^{12t}$ to find that $t \approx 15.3$ years