Exam 4 Review

1. \(3 \cdot 12 \cdot 6 \cdot 8 \cdot 4 = 6,912\)
2. \(\frac{6!}{6!2!3!10!6!3!4!9!2!4!2!2!3!}\)
3. \(\frac{2 \cdot 4 \cdot 3!}{5!} = \frac{48}{120} = \frac{2}{5}\)
4. \(P(9,4) = 3,024, P(10,3) = 720, P(4,2) = 12\)
5. \(C(9,4) = 126, C(10,3) = 120, C(4,2) = 6\)
6. \(C(100,7) = 1,601 \times 10^{10}\)
7. \(P(7,3) = 210\)
8. \(\frac{C(4,3)}{2^4} = \frac{1}{4}\)
9. \(\frac{2 \cdot 2!}{3!} = \frac{2}{3}\)
10. \(\frac{C(4,3)}{C(5,3)} = \frac{2}{5}\)
11. \(\frac{3 \cdot 2 \cdot 2 \cdot 1 \cdot 1}{5!} = \frac{1}{10}\)
12. \(\frac{2}{C(9,4)} = \frac{1}{63}\)
13. \(E(X) = (0.8)(30,000) + (0.2)(-10,000) = 22,000\)
14. \(E(X) = 8(4.00) - 10(1.20) = 20\) for all of the smoothies. He makes 10 total so the expected return on each smoothie is \(\frac{20}{10} = 2\).
15. \(E(X) = 50(-32) + 17(50) + 13(40) + 14(30) = 190\) for all of the tickets. He buys 50 total so the expected return for each ticket is \(\frac{190}{50} = 3.80\).
16. \(P(A \mid B) = \frac{1}{8}, P(B \mid A) = \frac{1}{4}\)
17. \(P(M \mid TV) = \frac{1}{40}\)
18. The two events are dependent since \(P(E \cap F) \neq P(E) \cdot P(F)\). That is, \(\frac{1}{8} \neq \frac{1}{2} \cdot \frac{3}{8}\).
38. \( \frac{4}{13} \cdot \frac{4}{13} = \frac{16}{169} \)

40. \( 2 \cdot \frac{5}{13} \cdot \frac{4}{13} = \frac{40}{169} \). We multiply by 2 because \( \frac{5}{13} \cdot \frac{4}{13} \) is the probability of drawing a white then a blue. We also have to account for the possibility of drawing a blue then a white. We could have written this instead as \( \frac{5}{13} \cdot \frac{4}{13} + \frac{4}{13} \cdot \frac{5}{13} = \frac{40}{169} \).

13. \( P(Fight|Mom) = 0.25, P(Fight|Me) = 0.25, P(Fight|Both) = 0.50. \)
\( P(Mom) = 0.60, P(Me) = 0.05, P(Both) = 0.03. \)

Find the probabilities that we fight and either of us is cranky like this:

\[
0.25 = P(Fight|Mom) = \frac{P(Fight \cap Mom)}{P(Mom)} = \frac{P(Fight \cap Mom)}{0.60} \implies P(Fight \cap Mom) = 0.15
\]

\[
0.25 = P(Fight|Me) = \frac{P(Fight \cap Me)}{P(Me)} = \frac{P(Fight \cap Me)}{0.05} \implies P(Fight \cap Me) = 0.0125
\]

\[
0.50 = P(Fight|Both) = \frac{P(Fight \cap Both)}{P(Both)} = \frac{P(Fight \cap Both)}{0.03} \implies P(Fight \cap Both) = 0.015
\]

Now our ultimate goal is to find \( P(Fight \cap Mom \text{ or } Me) \) So we use this formula:

\[
P(Fight \cap Mom \text{ or } Me) = P(Fight \cap Mom) + P(Fight \cap Me) - P(Fight \cap Both) = 0.15 + 0.0125 - 0.015 = 0.1475
\]

to conclude that my mom and I fight 14.75% of the time.

14. \( 1 - [P(x = 0) + P(x = 1) + P(x = 2)] = \)
\[
1 - [C(9, 0)(0.70)^0(0.30)^9 + C(9, 1)(0.70)(0.30)^8 + C(9, 2)(0.70)^2(0.30)^7] = 0.9957
\]