This is the format and directions that you will see on the cover page of the actual exam.

Name:

Discussion Section:

Discussion Instructor:

You may use a scientific calculator, but you may not use books, notes, graphing calculators, or your neighbors’ papers. Sign your name below to certify that you followed these instructions.

Signature:

Do all your work in the space provided on these sheets. If you need additional paper, attach it to these sheets.

On the multiple choice questions, clearly indicate the answer that you choose. If your selection is not clear, you will not earn any points for that problem.

Partial credit will be rewarded on the short answer problems. You will not earn credit for illogical, incorrect, or unsupported work, even if you miraculously arrive at the correct answer. If you are not certain how to do a problem, give it your best attempt so that you may earn some credit for moving in the right direction.

Circle your final answer on the short answer problems.

The exam will be graded out of 100 points. The point value for each problem is listed beside the problem number. There are 9 pages and 12 problems on the exam.

Good luck!
1. (10 points) Determine whether each of the following graphs is symmetric about the $x$-axis, the $y$-axis, the origin, or none of these. Show your work.

(a) $y^2 = x^2 + y^3$

Solution: $(-y)^2 = x^2 + (-y)^3$ is not the same as $y^2 = x^2 + y^3$, so the graph is not symmetric about the $x$-axis.

$y^2 = (-x)^2 + y^3$ is the same as $y^2 = x^2 + y^3$, so the graph is symmetric about the $y$-axis.

$(-y)^2 = (-x)^2 + (-y)^3$ is not the same as $y^2 = x^2 + y^3$, so the graph is not symmetric about the origin.

(b) $x^3 = xy$

Solution: $x^3 = x(-y)$ is not the same as $x^3 = xy$, so the graph is not symmetric about the $x$-axis.

$(-x)^3 = (-x)y$ is the same as $x^3 = xy$, so the graph is symmetric about the $y$-axis.

$(-x)^3 = (-x)(-y)$ is not the same as $x^3 = xy$, so the graph is not symmetric about the origin.
2. (8 points) Graph the inequality

\[ 3x - 5y < 15. \]

Be sure to label the axes and intercepts.

**Solution:** There is a dotted line passing through \((0, -3)\) and \((5, 0)\). The region to the left of the line is shaded. The axes should be labeled “‘x’” (horizontal) and “‘y’” (vertical).

3. (8 points) After a difficult exam, a professor took a survey of the number of hours students studied the day before the exam. The results were modeled by the function

\[ f(x) = -\frac{2}{3}x^2 + 16x, \]

where \(x\) is the number of hours studied the day before the exam and \(f(x)\) was the score earned on the exam. Assuming this model is correct, how many hours should a student have studied the day before the exam in order to maximize his or her score? What is the highest score one could achieve?

**Solution:** 12 hours, score of 96
4. (14 points) Consider the circle

\[ x^2 + y^2 - 3y - 4 = 0. \]

(a) Identify the center and radius.

**Solution:** center \((0, 3/2)\), radius \(5/2\)

(b) Graph the circle. Label the axes, the center, and four points on the circle.

**Solution:** The axes are labeled "x" (horizontal) and "y" (vertical). The center is labeled \((0, 3/2)\). The easiest four points to label are \((-5/2, 3/2)\), \((5/2, 3/2)\), \((0, 4)\), and \((0, -1)\).
5. (12 points) It costs Latisha $4.00 to buy supplies for a lemonade stand. She sells lemonade for $0.50 a glass.

(a) What will Latisha’s profit be if she sells 10 glasses of lemonade?

**Solution:** $1.00

(b) How many glasses of lemonade must Latisha sell in order to pay back the money she spent on supplies?

**Solution:** 8 glasses

(c) Write a linear function \( f(x) \) describing Latisha’s profit, where \( x \) is the number of glasses of lemonade sold.

**Solution:** \( f(x) = -4 + 0.5x \)

(d) Graph the linear function. Label the axes, the intercepts, and the points corresponding to the situations in (a) and (b). (Keep in mind that \( x \geq 0 \), so the domain is \([0, \infty)\).)

**Solution:** The graph is a ray starting at \((0, -4)\) and extending to the right through \((8, 0)\). The axes should be labeled “’x’” (horizontal) and “’y’” (vertical). Label the points \((0, -4)\), \((8, 0)\), and \((10, 1)\).
6. (12 points) Consider the quadratic function \( f(x) = -2x^2 - 4x \).

(a) Write the function in the form \( f(x) = a(x - h)^2 + k \).

\textbf{Solution:} \( f(x) = -2(x + 1)^2 + 2 \)

(b) What is the vertex of this parabola?

\textbf{Solution:} \((-1, 2)\)

(c) Graph the quadratic function. Label the axes, the vertex, and the intercepts.

\textbf{Solution:} The graph is a parabola opening downward, with vertex at \((-1, 2)\). The intercepts are \((-2, 0)\) and \((0, 0)\). The axes should be labeled ""x"" (horizontal) and ""y"" (vertical).

(d) What is the domain of this function? What is the range?

\textbf{Solution:} domain is all real numbers; range is \( \{f(x) : f(x) \leq 2\} \)
7. (6 points) Find the center and radius of the circle

\[(x + 7)^2 + (y - 4)^2 = 12.\]

(a) center \((-4, 7)\), radius 12
(b) center \((-4, 7)\), radius \(2\sqrt{3}\)
(c) center \((-7, 4)\), radius 12
(d) center \((-7, 4)\), radius \(2\sqrt{3}\)

Solution: d

8. (6 points) Find the length and the midpoint of the line segment connecting \((5, -2)\) to \((1, 4)\).

(a) midpoint \((3, 1)\), length \(2\sqrt{13}\)
(b) midpoint \((3, 1)\), length \(2\sqrt{5}\)
(c) midpoint \((2, -3)\), length \(2\sqrt{13}\)
(d) midpoint \((2, -3)\), length \(2\sqrt{5}\)

Solution: a
9. (6 points) The graph of \( f(x) = x^2 \) is shifted 2 units up and 3 units to the left. What is the new function?

(a) \( f(x) = (x - 3)^2 - 2 \)
(b) \( f(x) = (x + 2)^2 + 3 \)
(c) \( f(x) = (x - 2)^2 - 3 \)
(d) \( f(x) = (x + 3)^2 + 2 \)

**Solution:** d

10. (6 points) Find \( f(0) \) and \( f\left(\frac{1}{2}\right) \) if

\[
f(x) = \begin{cases} 
  x^2 + x & \text{if } x < 0 \\
  x + 3 & \text{if } x \geq 0
\end{cases}
\]

(a) \( f(0) = 0 \) and \( f\left(\frac{1}{2}\right) = \frac{7}{2} \)
(b) \( f(0) = 3 \) and \( f\left(\frac{1}{2}\right) = \frac{7}{2} \)
(c) \( f(0) = 0 \) and \( f\left(\frac{1}{2}\right) = \frac{3}{4} \)
(d) \( f(0) = 3 \) and \( f\left(\frac{1}{2}\right) = \frac{3}{4} \)

**Solution:** b
11. (6 points) Which line is perpendicular to $5x - 3y = 12$?
   (a) $-5x + 3y = 12$
   (b) $5x + 3y = 12$
   (c) $-3x + 5y = 12$
   (d) $3x + 5y = 12$

   \textbf{Solution: } d

12. (6 points) Consider the function $f(x) = \sqrt{x^2 - 9}$. What is the domain of this function?
   (a) $(-\infty, -3] \cup [3, \infty)$
   (b) $[-3, 3]$
   (c) $\{x : x \neq 3, x \neq -3\}$
   (d) $\{-3, 3\}$

   \textbf{Solution: } a