This is the format and directions that you will see on the cover page of the actual exam.

**Name:**

**Discussion Section:**

**Discussion Instructor:**

You may use a scientific calculator, but you may not use books, notes, graphing calculators, or your neighbors’ papers. Sign your name below to certify that you followed these instructions.

**Signature:**

Do all your work in the space provided on these sheets. If you need additional paper, attach it to these sheets.

On the multiple choice questions, clearly indicate the answer that you choose. If your selection is not clear, you will not earn any points for that problem.

Partial credit will be rewarded on the short answer problems. You will not earn credit for illogical, incorrect, or unsupported work, even if you miraculously arrive at the correct answer. If you are not certain how to do a problem, give it your best attempt so that you may earn some credit for moving in the right direction.

Circle your final answer on the short answer problems.

The exam will be graded out of 100 points. The point value for each problem is listed beside the problem number. There are 9 pages and 12 problems on the exam.

Here are some formulas that you might find useful.

\[ A = P(1 + r)^t \quad A = P \left(1 + \frac{r}{n}\right)^{nt} \quad A = Pe^{rt} \]
1. Consider the function \( f(x) = 1 - x^2 \).

(a) Is this function one-to-one? Why or why not?

**Solution:** no, because its graph does not pass the Horizontal Line Test

(b) Does this function have an inverse? If so, find it.

**Solution:** no, because it is not one-to-one

(c) Now consider the function \( f(x) = 1 - x^2 \) with domain \( \{x : x \geq 0\} \). Find \( f^{-1}(x) \).

**Solution:** \( f^{-1}(x) = \sqrt{1-x} \)

(d) What is the domain of \( f^{-1} \)? What is the range?

**Solution:**
domain: \( \{x : x \leq 1\} \)
range: \( \{y : y \geq 0\} \)
2. Consider the function \( f(x) = \frac{1}{3}x(x^2 - 4)^2 \).

(a) Find the roots of \( f(x) \), and express them as ordered pairs.

\[ \text{Solution: (0, 0), (2, 0), (-2, 0)} \]

(b) Find the \( y \)-intercepts of \( f(x) \), and express them as ordered pairs.

\[ \text{Solution: (0, 0)} \]

(c) Find the intervals where \( f(x) > 0 \) and the intervals where \( f(x) < 0 \).

\[ \text{Solution:} \]
\[ f(x) > 0 \text{ on } (0, 2) \cup (2, \infty) \]
\[ f(x) < 0 \text{ on } (-\infty, -2) \cup (-2, 0) \]

(d) Graph the function \( f(x) \). Label axes and intercepts.

\[ \text{Solution: Label the axes "x" (horizontal) and "y" (vertical). The inter-} \]
\[ \text{cepts were found in parts (a) and (b). The function is a smooth curve } \]
\[ \text{going through these points, satisfying the information from part (c).} \]
3. Solve the equation. \[ 16^{2x-1} = \left(\frac{1}{2}\right)^{x-3} \]

**Solution**: \[ x = \frac{7}{9} \]

4. Solve the equation. \[ \log(x^2) - \log(20 - x) = 1 \]

**Solution**: \( x = \{10, -20\} \)
5. (12 points) Consider the function \( f(x) = -|x| + 2 \).
   (a) Find \( f(-2) \) and \( f(5) \).

   **Solution:** \( f(-2) = 0, f(5) = -3 \)

   (b) Is this function even, odd, or neither? Show your work.

   **Solution:** even

   (c) Graph this function. Label axes, intercepts, and both points you found in part (a). (It might be helpful to first think about the graphs of \( g(x) = |x| \) and \( h(x) = -|x| \).)

   **Solution:** The graph is two rays. From \( x = -\infty \) to \( x = 0 \), there is a ray which passes through \((-2,0)\) and \((0,2)\). From \( x = 0 \) to \( x = \infty \), there is a ray which passes through \((0,2)\) and \((2,0)\). The general shape is an upside-down V, with a maximum at \((0,2)\). The axes should be labeled “\(x\)” (horizontal) and “\(y\)” (vertical). The points \((-2,0), (0,2), (2,0), \) and \((5,-3)\) should be labeled.
6. The number of bacteria present in a certain culture after \( t \) hours is given by the equation

\[ Q = Q_0 e^{0.34t}, \]

where \( Q_0 \) is the initial number of bacteria. How long will it take 300 bacteria to double? Round your solution to the nearest hundredth of an hour.

**Solution:** approximately 2.04 hours
7. Evaluate the logarithm to three decimal places. \( \log_3(21) \)
   (a) 0.361
   (b) 0.845
   (c) 2.771
   (d) 7.000

**Solution:** c

8. Consider the function \( f \circ g \), where \( f(x) = \frac{1}{x} \) and \( g(x) = \sqrt{x + 1} \). What is the domain of \( f \circ g \)?
   (a) \( \{ x | x = -1 \} \)
   (b) \( \{ x | x > -1 \} \)
   (c) \( \{ x | x \geq -1 \} \)
   (d) \( \{ x | x \neq -1 \} \)

**Solution:** b
9. (6 points) The function \( f(x) = |x| \) is stretched by a factor of 2, shifted 1 unit right and 3 units down. What is the equation of the new function?

(a) \( f(x) = 2|x - 1| - 3 \)
(b) \( f(x) = 2|x + 1| - 3 \)
(c) \( f(x) = |2x + 1| - 3 \)
(d) \( f(x) = |2x - 1| - 3 \)

Solution: a

10. (6 points) Find the composition \( f \circ g(x) \) when

\[
f(x) = x - 1 \quad \text{and} \quad g(x) = \sqrt{2x + 1}.
\]

(a) \( f \circ g(x) = \sqrt{2x} \)
(b) \( f \circ g(x) = \sqrt{2x + 1} - 1 \)
(c) \( f \circ g(x) = \sqrt{2x - 1} \)
(d) \( f \circ g(x) = \sqrt{2x + 1} \)

Solution: b
11. Find the total amount of money accumulated at the end of 8 years if $3,000 is invested at 5% interest, compounded four times a year.

(a) $4,432.37
(b) $4,475.47
(c) $3,313.46
(d) $4,464.39

Solution: d

12. Express this logarithm as the sum, difference, and/or product of simpler logarithmic quantities. Assume all variables represent positive real numbers.

\[ \log \left( \frac{\sqrt{x + y}}{z^3} \right) \]

(a) \( \log \left( \frac{1}{2} (x + y) - 3z \right) \)
(b) \( \frac{1}{2} \log(x) \log(y) - 3 \log z \)
(c) \( \frac{1}{2} \log(x + y) - 3 \log z \)
(d) \( \log \sqrt{x} + \log \sqrt{y} - 3 \log z \)

Solution: c