Math 1272
Midterm 3

Show all work for each problem so that partial credit may be given. Answers without supporting work will receive no credit. Clearly indicate your answers. Make sure that you have 6 problems on your test. A scientific calculator may be used.

1. (15)
2. (15)
3. (15)
4. (20)
5. (20)
6. (15)
Total (100)
1. (15 points) Given that the Taylor polynomial of degree 4 for \((4 + x)^{-1}\) is

\[
\frac{1}{4} - \frac{x}{16} + \frac{x^2}{64} - \frac{x^3}{256} + \frac{x^4}{1024},
\]

find the Taylor polynomial of degree 5 for \(\ln(4 + x)\).
2. (15 points) Show that the following series is convergent by using the comparison theorem and comparing it with the \( P \) series \( \sum \frac{1}{n^2} \).

\[
\sum_{n=1}^{\infty} \frac{\sin^2 n}{(2n+1)(2n-1)}
\]
3. (15 points) Find the Taylor polynomial $T_4(x)$ for $f(x) = e^{x-2} + \frac{1}{x-1}$ centered at $a = 2$. 
4. (20 points) Show that the alternating series is convergent.

\[
\sum_{n=1}^{\infty} (-1)^n \frac{n^2 + 2}{n^3(2n^4 + 7)}
\]
5. (20 points) For what values of \( x \) is the following power series convergent? divergent? Do not test endpoints.

\[
\sum_{n=0}^{\infty} \frac{(-1)^n (2n + 1)(3x)^n}{(5n^2 + 1)}
\]
6. (15 points) Suppose we use the finite sum \( \sum_{k=1}^{N} \frac{(-1)^{k+1}}{(3k+3)^2} \) to approximate the infinite sum \( \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(3k+3)^2} \). What is the smallest value of \( N \) (the smallest number of terms) that we may use to be sure that the error is less than \( 10^{-6} \)? Assume the hypothesis of the estimation theorem is satisfied.