Quiz 12 on Calculus II
Summer 2009

Name: ________________________________

You may use any resources, including the text, notes, or homework problems, and you are encouraged to work with other students or get outside help. However, you must write up your own solutions. In order to get full credit you must show all your steps.

1. Determine whether the series converges.

\[
\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n)}{n!}
\]
2. Evaluate the indefinite integral as an infinite series.

\[ \int x \cos(x^3) \, dx \]
3. Approximate \( f(x) = x^{2/3} \) by a Taylor polynomial of degree 3 about \( x = 1 \).
4. Find the Taylor series for the function \( f(x) = e^x \) centered at \( a = 3 \), assuming that the Taylor series exists.

5. Where does the power series converge? Where does it diverge? There is no need to check endpoints.

\[
\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt{n^2}}
\]
6. Determine whether the series converges.

\[ \sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{n^3} \]
7. Find the radius of convergence of the power series

\[ \sum_{n=1}^{\infty} n!(2x - 1)^n. \]

8. Using the fact that

\[ \frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n \text{ for } |x| < 1, \]

find a power series representation for \( f(x) = \frac{x}{9 + x^2}. \)
9. Find $T_3$, the Taylor polynomial of degree 3, for the function $f(x) = e^{x^2}$ about $a = 0$. 

10. Find a power series representation for the function

\[ f(x) = \ln(5 - x) \]

by first finding a power series for \( f(x) = \frac{-1}{5-x} \), then integrating. Find the radius of convergence for both power series.