Worksheet 9.3 Solutions

Summer 2009

1. Determine whether the following differential equations are separable.

(a) \( y' = (3y)(3x + 1) \) 

separable

(b) \( \frac{dy}{dx} = xe^{x+2y} \)

\( \frac{dy}{dx} = (xe^x)(e^{2y}) \) separable

(c) \( 7yy' = \ln x + 3 \)

\( y' = \left(\frac{1}{7y}\right)(\ln x + 3) \) separable

(d) \( \frac{dy}{dx} = 4y + x + 2 \)

not separable

(e) \( 4x + \frac{dy}{dx} = 3y^2 \)

not separable

(f) \( y' = 3x^2 + x^2y^3 \)

\( y' = (x^2)(3 + y^3) \) separable

2. A tank contains 2000 L of pure water. Brine that contains .03 kg of pure salt per liter of water enters the tank at a rate of 2 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. Set up a differential equation for the function \( S(t) \), the amount of salt in the tank at time \( t \). What is the initial value?

\[ \frac{dS}{dt} = .03 \cdot 2 - \frac{s}{2000} \cdot 2 \]

initial condition is \( S(0) = 0 \) since the tank holds pure water at \( t = 0 \)

3. A tank contains 5 L of pure water. Brine that contains .05 kg of salt per liter of water enters the tank at a rate of 1 L/min. Brine that contains .04 kg of salt per liter of water enters the tank at a rate of .2 L/min. The solution is kept thoroughly mixed and drains from the tank at a rate of 1.2 L/min. Set up a differential equation for the function \( S(t) \), the amount of salt in the tank at time \( t \).

\[ \frac{dS}{dt} = (.05 \cdot 1 + .04 \cdot .2) - \frac{s}{5} \cdot 1.2 \]

4. A vat with 200 gallons of beer contains 6% alcohol (by volume). Beer with 4% alcohol is pumped into the vat at a rate of 5 gal/min and the mixture is pumped out at the same rate. Set up an initial value problem for \( A(t) \), the amount of alcohol in the vat at time \( t \).

\[ \frac{dA}{dt} = .04 \cdot 5 - \frac{A}{200} \cdot 5 \]

initial condition is \( A(0) = .06 \cdot 200 = 12 \)